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RESEARCH MEMORANDUM

NUMERICAL SOLUTION OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW IN ROTATING COOLANT PASSAGES

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

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NUMERICAL SOLUTION OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW IN ROTATING COOLANT PASSAGES

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SUMMARY

A theoretical analysis was made of the air flow through the blade coolant passages in an air-cooled turbine rotor; one-dimensional flow was assumed. The simultaneous effects of area change, compressibility, wall friction, heat transfer, and rotation were included. A numerical method for solving the differential equations expressing the conservation of energy and momentum is presented for the general case in which the coefficients in the two equations are allowed to vary along the blade span. In addition, the variation of the combustion-gas effective temperature and relative velocity at the entrance to the rotor are considered in the analysis.

Tables of the several Mach number functions that appear in the differential equations are presented. The interval in the Mach number in these tables is small; the labor of interpolation is thereby minimized.

A numerical example is solved by use of the general solution of the differential equations. For the same example, a simplified solution of the energy equation is presented, in which mean constant values were used for the coefficients. The plotted results of the two solutions indicate that total temperature, Mach number, relative velocity, and static pressure of the coolant determined from the simplified solution deviated less than 2 percent from the corresponding values obtained for the general solution.

It was found that the general solution, in which the coefficients in the differential equations were treated as variables, should be employed for accurate determination of the spanwise blade-metal temperature distribution. The blade-metal temperature distribution may be approximately determined, however, from a simplified solution of the energy equation by utilizing the effective combustion-gas-temperature distribution and an approximate value of the variation in the ratio of outside to inside heat-transfer coefficient.

INTRODUCTION

Turbine-inlet temperatures higher than those encountered in current use offer possibilities for large improvements in the performance of aircraft gas turbines. For this reason, the net effect of different turbine-cooling methods on engine and airplane performance and means for utilizing higher inlet temperatures are being investigated at the NACA Lewis laboratory. An important phase of this work is the analysis of coolant flow through passages in the rotor blades.

In the air-cooled turbine, the cooling air may be introduced into the turbine disk near the center from some stage of the main compressor. The air then flows in a radial direction and enters a radial blade coolant passage at the base of the blades. The pressure and the temperature of the air flowing through the blade coolant passages are affected by heat transfer, wall friction, rotation of the turbine wheel, and changes in flow area along the coolant passage. The cooling air is often discharged into the working-fluid gas stream at the tips of the blades. An analysis of the air flow in the blade coolant passages is necessary in order to determine the pressure requirement for the cooling air and the effect of the coolant on the performance of the entire engine. A method for computing the pertinent flow characteristics of the cooling air within the blade coolant passages is presented herein.

Considerable research has been done on the ideal case of one-dimensional gas flow through stationary passages. Some of this work is summarized in reference 1.

Simplified solutions valid for low velocities (Mach numbers less than 0.4) and moderate heat-transfer rates (temperature differences between the air and the wall less than 300° F) are presented in references 2 and 3. Experimental work presented in reference 4 shows that for higher Mach numbers and higher heat-transfer rates, the simplified solution gave only very rough approximations. Even for the simple case (one-dimensional flow through stationary passages), no exact closed-form solution has been found for the momentum equation. References 5 and 6 present sets of working charts that facilitate the determination, without individual integration, of the pressure variations of a compressible fluid flowing through heat-exchanger passages in the specified case wherein heat is added to the air stream by the passage walls, which are at a constant temperature throughout their length. It is stated in reference 6 that similar methods could be used for the case in which the rate of heat input along the passage length is constant.

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A one-dimensional analysis of gas flow in a stationary passage is presented in reference 1; the simultaneous effects of area change, wall friction, drag of internal bodies, generalized body forces, external heat exchange, chemical reaction, change of phase, injection of gases, and changes in molecular weight and specific heat are considered. The analysis of reference 1 can be applied to the flow through the rotating blade coolant passages by replacing the generalized body force of reference 1 by the centrifugal forces due to the rotation of the blade coolant passage. In this connection, the differential equations expressing the conservation of energy and momentum are developed in forms that are applicable to the present problem. Methods for the simultaneous numerical solution of these two differential equations are presented.

The simultaneous solution of the energy and momentum equations from the tip of the blade to the root permits the determination of static pressure, total temperature, and velocity distributions of the coolant along the blade coolant passage. In addition, the spanwise blade-metal temperature distribution may be determined. Recommendations are also made for determining the heat-transfer coefficients, the friction coefficient, and the recovery coefficient, inasmuch as these coefficients appear in the differential equations. A numerical example is presented to demonstrate the application of the methods developed. This example is also solved by using two closed-form solutions of the energy equation and the results are compared. Also a simplified method of determining blade-metal temperatures is given. Tables are presented in order to facilitate the computations; these tables require less interpolating time than those of reference 7 and contain additional Mach number functions not given in reference 7.

ANALYSIS

The principal assumptions made are listed and discussed. The basic physical equations and the working equations derived from them for this application are also given as well as a numerical method for solving the differential equations and the solution of a numerical example to illustrate the method.

A schematic sketch of a hollow turbine blade mounted on the turbine disk is shown in figure 1(a). The analytical methods presented refer to the coolant flow in the section of the blade between the radii r_h and r_T. The remaining three sketches in figure 1 indicate possible hollow-blade cross sections to which the present analysis can be applied.

The velocity diagram of the coolant at radius r is superimposed on the sketch of a mounted hollow blade in figure 2(a). The velocity W referred to in the following analysis is the velocity of the coolant relative to the blade in the radial direction. The total temperatures employed are relative to the blade.

In a turbine blade rotating at a high angular velocity, the coolant flow is affected by rotation, change in flow area along the blade coolant passage, wall friction, and heat transfer from the hot blade walls to the coolant. The inside and outside perimeters of the blade as well as the coolant flow area vary, in general, along the blade span. The velocity of the combustion gas relative to the blades may vary as much as 100 percent along the blade span. All these variations cause changes in the Reynolds numbers of the combustion-gas flow and coolant flow; variations in the inside and outside heat-transfer coefficients along the blade span therefore result.

If the spanwise blade-metal temperature distribution is required, all these variations along the blade must be considered in the solution of the differential equations describing the coolant flow. In order to account for all the variations, a numerical solution is presented for the differential equations. If, however, an accurate blade-metal temperature distribution is not required, a closed-form solution of the energy equation may be employed without appreciably changing the computed distribution of temperature, pressure, velocity, density, and Mach number of the coolant.

Assumptions

In order to simplify the analysis, the following assumptions are made: These assumptions are listed and then discussed.

1. The coolant flow is one-dimensional and in the radial direction; that is, the flow properties are constant in any cylinder perpendicular to the radius.

2. Heat conduction spanwise along the blade to the rim is negligible.

3. Heat transferred by radiation (from the stator to the rotor) is negligible.

4. Inlet effects on heat transfer are negligible.

5. Thermal resistance of the blade wall is negligible.

The variation of pressure, temperature, velocity, and density across a section of the coolant passage is small enough in current turbine designs to permit assumption 1. For example, in the case of the Jumo-004 hollow-blade turbine, Stodola's formula for the velocity variation (reference 8) gives a variation across the passage of about 10 feet per second, which corresponds to a velocity variation of about 1 to 3 percent. The pressure variations would be of the same order of magnitude. Velocity variations across the passage could conceivably be produced by natural convection; if these velocities are compared with the forced velocities by comparing in a typical case the heat-transfer coefficients due to natural and forced convection, variations of about 10 percent appear possible. The largest deviation from an average value would thus not exceed 5 percent and a one-dimensional calculation would give the radial velocity trend with sufficient accuracy for many purposes.

The parameter that measures the relative importance of conduction to the rim is shown in reference 9 as

$$\zeta = (r - r_h)\sqrt{\frac{H_o l_o + H_i l_i}{k_B\, A_B}}$$

The spanwise blade-metal temperature distributions for certain air-cooled hollow blades, which have been studied, indicate that conduction to the rim is negligible beyond a point 0.8 inch from the rim. The value of ζ at this point is between 3 and 4. For the air-cooled hollow blade of reference 10, the value of ζ for negligible conduction to the rim is 3.4 and occurs at a point 10 percent of the blade span from the root (0.4 in.). Equation (10) of reference 11 (p. 232) indicates that when ζ is equal to or greater than 4, the temperature difference between the combustion gas and the blade metal is changed less than 1.8 percent by conduction to the rim. Assumption 2 thus appears to be justified when ζ is equal to or greater than 3.5 at the point in question. In the numerical example presented herein, ζ is equal to 3.5 at a point 0.7 inch (19 percent of the blade span) from the root.

Assumption 3 has been studied in an unpublished hollow-blade calculation; the effect of radiation was found to be very small for stator temperatures up to about 2000° F. Because of the very small contribution of radiation to the total heat transferred at 2000° F, radiation effects may probably be neglected without sizeable error for stator temperatures as high as 3000° F.

Assumption 4 is supported by unpublished data on heat transfer in a tube. 1199

Assumption 5 is customarily used when a gas film exists on each side of a thin metal wall.

Basic Physical Equations

Four basic physical equations are available for the determination of the distributions of Mach number, static pressure, total temperature, and the velocity of the coolant along the blade coolant passage. The equation of state and the continuity equation, respectively, are

$$p = \rho g R T \tag{1}$$

$$w = \rho g A W \tag{2}$$

(All symbols are defined in appendix A.)

The general form of the energy equation is

$$dq + \frac{F_b\, dr}{J} = c_p\, dT + d\, \frac{W^2}{2gJ} \tag{3}$$

as in reference 1 and the general form of the momentum equation is

$$\frac{dp}{\rho g} - F_b\, dr + \frac{W\, dW}{g} + \frac{dp_{fr}}{\rho g} = 0 \tag{4}$$

as in reference 11 (p. 117).

Working Form of Differential Equations

The independent variable r may conveniently be changed to a dimensionless number y defined so as to vary from a value of zero at the blade tip to a value of unity at the blade root. Thus

$$y = (1/b)(b + r_h - r) \tag{5}$$

The main dependent variables are the total temperature of the coolant T'' and the Mach number of the coolant M; both are referred to the rotating passage. The dependent variables are defined in the usual manner.

$$T'' = T + \frac{W^2}{2Jgc_p} \tag{6}$$

$$M^2 = \frac{W^2}{\gamma gRT} \tag{7}$$

The distributions of Mach number, static pressure, total temperature, coolant velocity, and blade-metal temperature along the blade coolant passage are shown in appendixes B and C to depend on the solution of the differential equations of energy and momentum. The working form of the energy equation is

$$\frac{dT''}{dy} = -\frac{bH_{o,w}l_o}{wc_p(1+\lambda)}\left\{T_{g,e} - T''\left[1 - (1 - A_i)\, I_{T,e}\right]\right\} - \frac{b^2\omega^2}{Jgc_p}(1-y) - \frac{br_h\omega^2}{Jgc_p} \tag{8}$$

and of the momentum equation is

$$\frac{dM^2}{dy} = \frac{I_T}{T''}\frac{dT''}{dy} - I_f\left[\frac{4fb}{D_h} - \frac{I_R}{T''}\frac{2\omega^2\, r_h b}{gR}\left(1 + \frac{b}{r_h} - y\,\frac{b}{r_h}\right)\right] + \frac{I_A}{A}\frac{dA}{dy} \tag{9}$$

where

$$I_{T,e} = \frac{\frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2} \tag{10}$$

$$I_R = \frac{1 + \frac{\gamma-1}{2} M^2}{\gamma M^2} \tag{11}$$

$$I_A = - \frac{2M^2\left(1 + \frac{\gamma-1}{2} M^2\right)}{1 - M^2} \tag{12}$$

$$I_f = \frac{\gamma M^4\left(1 + \frac{\gamma-1}{2} M^2\right)}{1 - M^2} \tag{13}$$

$$I_T = \frac{M^2\left(1 + \gamma M^2\right)\left(1 + \frac{\gamma-1}{2} M^2\right)}{1 - M^2} \tag{14}$$

and Λ_i is the recovery coefficient defined as

$$\Lambda_i = \frac{T_e - T}{T'' - T} \tag{15}$$

where T_e is the adiabatic wall temperature, that is, the temperature assumed by the wall in the absence of heat transfer between the wall and the moving fluid.

All the functions have been calculated and are presented in table I. Some of these values have been published in reference 7. In the present tables, some new functions have been added to those of reference 7. An additional specific heat ratio (1.37) has been computed in order to simplify interpolation for varying air temperatures. The tabular interval in the Mach number has been reduced also to facilitate interpolation.

In general, equations (8) and (9) must be solved simultaneously because the Mach number occurs in the energy equation (8) and the total temperature occurs in the momentum equation (9). In many instances, especially when Λ_1 is close to unity, the term involving the Mach number in the energy equation is quite small, less than 2 percent of unity. In such cases, this term is often neglected in order to permit the energy equation to be solved independently of the momentum equation. This solution is presented in the section APPROXIMATE CLOSED-FORM SOLUTION OF ENERGY EQUATION.

Determination of Flow Characteristics

When the differential equations (8) and (9) have been solved for the distributions of Mach number and coolant total temperature along the blade coolant passage, the static-pressure and coolant-velocity distributions may be determined as in reference 1 from the equations

$$\frac{p}{p_T} = \frac{A_T}{A}\frac{M_T}{M}\sqrt{\frac{T''}{T''_T}\left(\frac{1+\frac{\gamma-1}{2}M_T^2}{1+\frac{\gamma-1}{2}M^2}\right)} \tag{16}$$

$$\frac{W}{W_T} = \frac{M}{M_T}\sqrt{\frac{T''}{T''_T}\left(\frac{1+\frac{\gamma-1}{2}M_T^2}{1+\frac{\gamma-1}{2}M^2}\right)} \tag{17}$$

Equation (16) is obtained from the combination of equations (1), (2), (6), and (7). Equation (17) is obtained by eliminating T from equations (6) and (7) and substituting $R\gamma/(\gamma-1)$ for Jc_p.

NUMERICAL METHOD OF SOLUTION

The numerical methods are developed in reference 12. In order to adapt these methods to the specific problem considered, equal increments of length along the blade coolant passage are designated by the stations 0, 1, 2, . . . n, beginning at the blade tip. The distances to each of these stations are designated by $y_0 (y_0 = 0)$, $y_1, y_2, \ldots y_n$, respectively. A simultaneous solution of equations (8) and (9) is required; the procedure for solving both equations is identical. The total-temperature distribution, which is obtained from the energy equation, is nearly linear, whereas the Mach number distribution, which is obtained from the momentum equation, is not; consequently, all the refinements required for a function with a rapidly changing slope must be utilized in the solution of the momentum equation. The details of the solution of the momentum equation are therefore presented. The energy equation may be solved in a similar manner by employing as many of the refinements as are required.

The solution is initiated by determining the Mach number at the tip from the relation

$$I_c = \gamma M^2 \left(1 + \frac{\gamma-1}{2} M^2\right) = \frac{w^2 R T''}{A^2 g p^2} \qquad (18)$$

which is derived in appendix D. All quantities on the right side of equation (18) are assumed to be known and the Mach number at the tip of the blade can be found by reference to table I.

The insertion of the initial value of M^2 (when $y = 0$) in equation (9) yields $(dM^2/dy)_0$. The value of M at the station 1/2 is computed from the equation

$$(M^2)_{1/2} = (M^2)_0 + \left(\frac{dM^2}{dy}\right)_0 \left(y_{1/2} - y_0\right) \qquad (19)$$

The value of $(M^2)_{1/2}$ permits the computation of $\left(\frac{dM^2}{dy}\right)_{1/2}$ from equation (9) using the values of I_A, I_f, I_T, and I_R from table I and the values of A, T'', dT''/dy, D_h, f, and $dA/A\,dy$

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corresponding to $y_{1/2}$. The simultaneous step-by-step solution of the energy equation yields T'' and dT''/dy.

The value of $(M^2)_1$ is then computed from the equation

$$(M^2)_1 = (M^2)_0 + \left(\frac{dM^2}{dy}\right)_{1/2} \left(y_1 - y_0\right) \tag{20}$$

As before, the value of $(dM^2/dy)_1$ is computed by inserting $(M^2)_1$ in equation (9).

Differences will hereinafter be employed for calculating quantities necessary in the numerical integration. The following equations indicate the method for computing the required differences at any station n:

$$\Delta_1 \left(\frac{dM^2}{dy}\right)_n = \left(\frac{dM^2}{dy}\right)_n - \left(\frac{dM^2}{dy}\right)_{n-1} \tag{21}$$

$$\Delta_2 \left(\frac{dM^2}{dy}\right)_n = \Delta_1 \left(\frac{dM^2}{dy}\right)_n - \Delta_1 \left(\frac{dM^2}{dy}\right)_{n-1} \tag{22}$$

$$\Delta_3 \left(\frac{dM^2}{dy}\right)_n = \Delta_2 \left(\frac{dM^2}{dy}\right)_n - \Delta_2 \left(\frac{dM^2}{dy}\right)_{n-1} \tag{23}$$

At station $n+1$, the values of $(dM^2/dy)_{n+1}$ and $(M^2)_{n+1}$ are obtained as follows:

It is important to have the numerical work systematically arranged. The values of the different functions of dM^2/dy for the stations 0, 1, 2, . . . n may be conveniently arranged as in the following table of differences:

y	$\frac{dM^2}{dy}$	Δ_1	Δ_2	Δ_3
y_0	$\left(\frac{dM^2}{dy}\right)_0$			
y_1	$\left(\frac{dM^2}{dy}\right)_1$	$\Delta_1 \left(\frac{dM^2}{dy}\right)_1$		
y_2	$\left(\frac{dM^2}{dy}\right)_2$	$\Delta_1 \left(\frac{dM^2}{dy}\right)_2$	$\Delta_2 \left(\frac{dM^2}{dy}\right)_2$	
y_3	$\left(\frac{dM^2}{dy}\right)_3$	$\Delta_1 \left(\frac{dM^2}{dy}\right)_3$	$\Delta_2 \left(\frac{dM^2}{dy}\right)_3$	$\Delta_3 \left(\frac{dM^2}{dy}\right)_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
y_n	$\left(\frac{dM^2}{dy}\right)_n$	$\Delta_1 \left(\frac{dM^2}{dy}\right)_n$	$\Delta_2 \left(\frac{dM^2}{dy}\right)_n$	$\Delta_3 \left(\frac{dM^2}{dy}\right)_n$

This table is formed by subtracting each number from the one immediately beneath it and setting the remainder opposite the minuend in the column to the right.

A similar table of differences may be constructed for the Mach number at the several stations. Third-order differences for either dM^2/dy or M will change slowly if the interval $y_{n+1} - y_n$ is sufficiently small. The table of differences in which the third-order differences change more regularly should be noted. The value of the argument of this table at station $n+1$ may be estimated by inference from the difference table. The third-order difference is first inferred at station $n+1$ from the values above it. The inferred difference is then added to the last entry in the second difference column. The resulting sum is added to the last entry in the first difference column, which is finally added to the last entry in the column for the tabulated values of the argument. The resulting value is the trial value of the argument.

The next step is to compute $\left(M^2\right)_{n+1}$. This term is found by the formula

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$$\left(M^2\right)_{n+1} - \left(M^2\right)_n = \left(y_{n+1} - y_n\right)\left[\left(\frac{dM^2}{dy}\right)_{n+1} - \frac{1}{2}\Delta_1\left(\frac{dM^2}{dy}\right)_{n+1} - \frac{1}{12}\Delta_2\left(\frac{dM^2}{dy}\right)_{n+1} - \frac{1}{24}\Delta_3\left(\frac{dM^2}{dy}\right)_{n+1} \cdot \cdot \cdot\right] \tag{24}$$

This value of $\left(M^2\right)_{n+1}$ is used in equation (9) to compute $(dM^2/dy)_{n+1}$. If this result differs considerably from the trial value of $(dM^2/dy)_{n+1}$ found by inference, the differences are recalculated and equation (24) used again. The value of $\left(M^2\right)_{n+1}$ seldom needs to be corrected more than once.

The values of M^2 and dM^2/dy at stations 2 and 3 are found just as those for station n+1 except that, for the step from 1 to 2, Δ_2 and Δ_3 are unknown and, for the step from 2 to 3, Δ_3 is unknown. Terms containing these differences are omitted. It is thus advisable at this stage, that is, when approximate values are known for the stations 0, 1, 2, and 3, to check the values of $\Delta_1 M_1^2$, $\Delta_1 M_2^2$, and $\Delta_1 M_3^2$ by the following equations:

$$\Delta_1 M_1^2 = \Delta y\left[\left(\frac{dM^2}{dy}\right)_3 - \frac{5}{2}\Delta_1\left(\frac{dM^2}{dy}\right)_3 + \frac{23}{12}\Delta_2\left(\frac{dM^2}{dy}\right)_3 - \frac{3}{8}\Delta_3\left(\frac{dM^2}{dy}\right)_3\right] \tag{25}$$

$$\Delta_1 M_2^2 = \Delta y\left[\left(\frac{dM^2}{dy}\right)_3 - \frac{3}{2}\Delta_1\left(\frac{dM^2}{dy}\right)_3 + \frac{5}{12}\Delta_2\left(\frac{dM^2}{dy}\right)_3 + \frac{1}{24}\Delta_3\left(\frac{dM^2}{dy}\right)_3\right] \tag{26}$$

$$\Delta_1 M_3^2 = \Delta y\left[\left(\frac{dM^2}{dy}\right)_3 - \frac{1}{2}\Delta_1\left(\frac{dM^2}{dy}\right)_3 - \frac{1}{12}\Delta_2\left(\frac{dM^2}{dy}\right)_3 - \frac{1}{24}\Delta_3\left(\frac{dM^2}{dy}\right)_3\right] \tag{27}$$

and revise M_1^2, M_2^2, M_3^3 in the tables.

With the revised values of M^2, the corresponding values of dM^2/dy are recomputed from equation (9) and the difference tables are corrected.

If in the course of the computations the differences of the highest order employed become very small, the interval between the values of y may be doubled. In order to accomplish the increase in the interval, the values of dM^2/dy corresponding to the alternate values of y employed in the last steps of the computation are omitted. A new difference table is constructed employing the remaining values of dM^2/dy. The new differences may then be employed in the continuation of the computation.

If, however, the differences of the highest order become very large or if these differences vary irregularly, the interval between the values of y employed must be decreased to one-half or one-third the original value. If the last line of differences $\Delta_i\,(dM^2/dy)_n$ (where $i = 1$, 2, and 3) in which the highest-order difference is still small occurs at a value of $y = y_n$, this line of differences must be replaced by a new line of differences $\Delta_{i,r}\,(dM^2/dy)_n$ (where $i = 1$, 2, and 3) corresponding to the reduced interval but applicable to the same station y_n. The computation of the value of $\Delta_{i,r}$ corresponding to the one-half interval from the value of Δ_i is accomplished by the following equations:

$$\left.\begin{aligned}
\Delta_{1,r}\left(\frac{1}{2}\right) &= \frac{1}{2}\,\Delta_1 + \frac{1}{8}\,\Delta_2 + \frac{1}{16}\,\Delta_3 + \ldots \\
\Delta_{2,r}\left(\frac{1}{2}\right) &= \frac{1}{4}\,\Delta_2 + \frac{1}{8}\,\Delta_3 + \ldots \\
\Delta_{3,r}\left(\frac{1}{2}\right) &= \frac{1}{8}\,\Delta_3 + \ldots
\end{aligned}\right\} \qquad (28)$$

The following equations are available for computing $\Delta_{i,r}$ for the one-third interval from Δ_i:

$$\left.\begin{aligned}
\Delta_{1,r}\left(\frac{1}{3}\right) &= \frac{1}{3}\,\Delta_1 + \frac{1}{9}\,\Delta_2 + \frac{5}{81}\,\Delta_3 \\
\Delta_{2,r}\left(\frac{1}{3}\right) &= \frac{1}{9}\,\Delta_2 + \frac{2}{27}\,\Delta_3 \\
\Delta_{3,r}\left(\frac{1}{3}\right) &= \frac{1}{27}\,\Delta_3
\end{aligned}\right\} \tag{29}$$

In a calculation initiated at the blade tip and proceeding toward the blade root, the interval can usually be doubled several times, even though small intervals are initially needed because of a high exit Mach number $(M > 0.7)$. If the exit Mach number is less than 0.5 and the area changes are small, large intervals (0.1 or 0.2) can be used throughout the passage for $y_{n+1} - y_n$. The intervals used in the numerical example should be taken only as a rough guide. The previously mentioned criterions should be applied in each case. This choice of interval is further discussed in the example.

NUMERICAL EXAMPLE OF GENERAL SOLUTION

The details of a numerical example for a finned blade are presented in order to illustrate the procedure to be followed in the simultaneous solution of the energy equation (8) and the momentum equation (9). The radial variations of $T_{g,e}$, W_g, c_p, γ, H_o, H_i, D_h, and the other blade dimensions are considered in the solution.

Variation Along Blade Span of $H_o l_o$ and $1 + \lambda$

Inspection of equation (8) shows that the coefficient of the principal temperature term involves the factors $H_o l_o$ and $1 + \lambda$ where

$$\lambda = \frac{H_{o,w}l_o}{H_{i,w}l_i} = \frac{H_{o,w}l_o}{H_{i,f}l_i}$$

In this case, $H_{i,w} = H_{i,f}$ because the blade studied has cooling fins inside the passage. Each of the four members of the ratio λ varies along the blade span. The heat-transfer coefficients $H_{o,w}$ and $H_{i,w}$ depend on the combustion-gas, cooling-air, and blade-metal temperatures as well as the position along the blade span; they cannot be tabulated until the pertinent temperatures are determined. The correlations employed to determine $H_{o,w}$ and $H_{i,w}$ depend on the local Reynolds number. Because the local Reynolds number varies along the blade span, the values of $H_{o,w}$ and $H_{i,w}$ must be determined at each station. The effect of the Reynolds number variation along the blade span is accounted for in the computation by calculating the reference values $\overline{H}_{o,w}\overline{l}_o$ and $\overline{H}_{i,w}\overline{l}_i$ that correspond to the actual values of the two parameters near the midpoint of the span and by multiplying these two reference values by the appropriate functions of temperature and geometry ratios. In order to make the variation calculations dimensionless, reference values were used. The ratios involved then approach unity and all the invariant factors cancel.

In the case of the outside heat-transfer coefficient, the correlation employed is determined from data obtained at the NACA Lewis laboratory (See appendix E).

$$\frac{H_{o,w}D_{h,o}}{k_{g,w}} = 0.75 \left(\frac{\rho_{g,w}W_g D_{h,o}}{\mu_{g,w}}\right)^{0.53} \left(Pr_{g,w}\right)^{1/3} \tag{30}$$

Now

$$\rho_{g,w} = \frac{p_{ex}}{gRT_B} \tag{31}$$

and

$$D_{h,o} = \frac{l_o}{\pi} \tag{32}$$

For a limited range

$$k_{g,w} \propto \mu_{g,w} \propto T_B^{0.7} \tag{33}$$

When equation (30) is divided by a similar equation written in terms of reference values and when equations (31) to (33) are employed, the following relation is obtained:

$$\frac{H_{o,w}l_o}{\overline{H}_{o,w}\overline{l}_o} = \left(\frac{l_oW_g}{\overline{l}_o\overline{W}_g}\right)^{0.53}\left(\frac{\overline{T}_B}{T_B}\right)^{0.201}\left(\frac{Pr_{g,w}}{\overline{Pr}_{g,w}}\right)^{1/3}$$

Over the usual range of temperatures from the root to the tip of the blade, the ratio $(Pr_{g,w}/\overline{Pr}_{g,w})^{1/3}$ differs from unity by only a few tenths of one percent and can therefore be omitted. The final form of the equation is

$$\frac{H_{o,w}l_o}{\overline{H}_{o,w}\overline{l}_o} = \left(\frac{l_oW_g}{\overline{l}_o\overline{W}_g}\right)^{0.53}\left(\frac{\overline{T}_B}{T_B}\right)^{0.201} \tag{34}$$

The first factor on the right side of equation (34) may be tabulated before the simultaneous solution is initiated, but the second factor must be tabulated as the blade-metal temperatures become available.

For the inside heat-transfer coefficient, the correlation of reference 13 is employed. (In this reference all bulk properties of the air are evaluated at the total temperature.)

$$\frac{H_{1,w}\,D_h}{k_w} = 0.019\left(\frac{\rho_w W D_h}{\mu_w}\right)^{0.8} \tag{35}$$

The presence of fins in the blade coolant passage may be taken into account by adapting the following finned-cylinder relation (reference 14) to the present case by making the radius infinite:

$$H_f = \frac{H_{1,w}}{m+\tau}\left(\frac{2\ \tanh\varphi\,L_f}{\varphi} + m\right) \tag{36}$$

The following relation is obtained for $H_f l_1$ in a manner similar to the derivation of equation (34). (See appendix F.)

$$\frac{H_f l_1}{\overline{H}_f \overline{l}_1} = \frac{l_1}{\overline{l}_1}\,\frac{\overline{A}}{A}\left(\frac{l_f}{\overline{l}_f}\right)^{0.2}\left(\frac{T''}{\overline{T}''}\right)^{0.7}\left(\frac{\overline{T}_B}{T_B}\right)^{0.56}\left(\frac{m + \dfrac{2\ \tanh\varphi\,L_f}{\varphi}}{m + \dfrac{2\ \tanh\overline{\varphi}\,\overline{L}_f}{\overline{\varphi}}}\right) \tag{37}$$

Because

$$\varphi = \sqrt{\frac{2\ H_{1,w}}{k_B\ \tau}}$$

$$\frac{\varphi\,L_f}{\overline{\varphi}\,\overline{L}_f} = \frac{L_f}{\overline{L}_f}\left(\frac{l_f}{\overline{l}_f}\right)^{0.1}\left(\frac{\overline{A}}{A}\right)^{0.5}\left(\frac{T''}{\overline{T}''}\right)^{0.35}\left(\frac{\overline{T}_B}{T_B}\right)^{0.28} \tag{38}$$

The right sides of equations (34) and (38) may thus be expressed as the product of the following two factors: the geometry factor G, which is dependent on the blade geometry, and the temperature-ratio factor θ, which is dependent on the temperature distributions. The final factor of equation (37), the fin factor, contains both geometric and temperature terms and must therefore be evaluated step by step as the temperatures become available. These factors for determining the heat-transfer parameters in terms of reference values are presented in the following table:

Position	Heat-transfer parameter	Geometry factor, G	Fin factor	Temperature factor, θ
Outside o	$\frac{H_{o,w} l_o}{\overline{H}_{o,w} \overline{l}_o}$	$\left(\frac{l_o W_g}{\overline{l}_o \overline{W}_g}\right)^{0.53}$		$\left(\frac{\overline{T}_B}{T_B}\right)^{0.201}$
Fin f	$\frac{H_f l_i}{\overline{H}_f \overline{l}_i}$	$\frac{l_i}{\overline{l}_i} \frac{\overline{A}}{A} \left(\frac{l_f}{\overline{l}_f}\right)^{0.2}$	$\dfrac{m + \frac{2 \tanh \varphi L_f}{\varphi}}{m + \frac{2 \tanh \overline{\varphi L_f}}{\overline{\varphi}}}$	$\left(\frac{T''}{\overline{T}''}\right)^{0.7} \left(\frac{\overline{T}_B}{T_B}\right)^{0.56}$
Inside i	$\frac{\varphi L_f}{\overline{\varphi L_f}}$	$\frac{L_f}{\overline{L}_f} \left(\frac{l_f}{\overline{l}_f}\right)^{0.1} \left(\frac{\overline{A}}{A}\right)^{0.5}$		$\left(\frac{T''}{\overline{T}''}\right)^{0.35} \left(\frac{\overline{T}_B}{T_B}\right)^{0.28}$

Assumed Conditions for Numerical Example

A blade configuration similar to that depicted in figure 1(d) was employed in the numerical example. The variation of coolant flow area, hydraulic diameter (inside), perimeters, mean half-width of fins, and area-change parameter $\frac{1}{A}\frac{dA}{dy}$ along the blade span is presented in figures 3(a) to 3(e). The turbine is assumed to be designed for free-vortex flow. The theoretical variation, along the blade span, of the combustion-gas velocity relative to the blades at the entrance to the rotor is presented in figure 3(f). A typical effective combustion-gas-temperature distribution along the blade span, which will be employed in the computation, is shown in figure 3(g). The variation of c_p and γ for the cooling air with temperature is plotted in figures 4 and 5, respectively. In figure 6, the variation of the friction coefficient with the blade-coolant-passage Reynolds number is presented. 1199

In order to compute the reference values of the inside and outside heat-transfer coefficients, the blade dimensions corresponding to the midpoint of the span ($y = 0.5$) are employed. The reference temperature for the combustion-gas properties $\bar{T}_B$ (reference 16) is assumed to be 1300° R and for the cooling air $\bar{T}''$ is assumed to be 900° R. The reference values are arbitrary, but are selected to be as near the middle of the probable range of values as possible. The data required for the numerical example are as follows:

$\bar{A}$	0.00041 (sq ft)
A_T	0.000194 (sq ft)
b	0.3 (ft)
$\bar{D}_h$	0.00668 (ft)
f	0.0065
k_B	0.00417 Btu/(sec)(ft)(°F)
$\bar{k}$	0.657×10^{-5} Btu/(sec)(ft)(°F)
$\bar{k}_{g,w}$	0.82×10^{-5} Btu/(sec)(ft)(°F)
$\bar{L}_f$	0.0055 (ft)
$\bar{l}_f$	0.2505 (ft)
$\bar{l}_i$	0.1295 (ft)
$\bar{l}_o$	0.262 (ft)
m	0.0050 (ft)
$(\overline{Pr}_{g,w})^{1/3}$	0.876
$\bar{p}_{ex}$	5000 (lb/sq ft)

p_T	5000 (lb/sq ft)
R	53.3 (ft-lb)/(lb)(°R)
r_h	1.117 (ft)
$\bar{T}_B$	1300° R
$\bar{T}''$	900° R
T''_T	1160° R
$\bar{W}_g$	838 (ft/sec)
w	0.01689 (lb/sec)
Λ	0.9
$\bar{\mu}$	0.558x10^{-6} slug/(ft)(sec)
$\bar{\mu}_{g,w}$	0.695x10^{-6} slug/(ft)(sec)
τ	0.0025 (ft)
ω	796 radians/(sec)

In evaluating the gas properties, a fuel-air ratio of 0.02 and a hydrogen-carbon ratio of 0.186 were assumed.

Calculation of Reference Values of $H_{o,w}l_o$ and $1 + \lambda$

Calculation of $\bar{H}_{o,w}\bar{l}_o$. - The reference value $\bar{H}_{o,w}\bar{l}_o$ is calculated corresponding to the reference temperature

$$\bar{T}_B = 1300^{\circ} \text{ R}$$

the blade dimensions employed are for

$$y = 0.5$$

The computations are as follows:

$$\bar{\rho}_{g,w} = \frac{\bar{p}_{ex}}{gR\bar{\bar{T}}_B} = \frac{5000}{(32.17)(53.3)(1300)} = 0.00224 \text{ (slug/cu ft)}$$

$$\bar{D}_{h,o} = \frac{\bar{l}_o}{\pi} = \frac{0.262}{\pi} = 0.0834 \text{ (ft)}$$

$$\overline{Re}_o = \frac{\bar{\rho}_{g,w}\bar{D}_{h,o}\bar{W}_g}{\bar{\mu}_{g,w}} = \frac{(0.00224)(0.0834)(838)}{0.695 \times 10^{-6}} = 225{,}000$$

$$\overline{Nu}_o = 0.75\ (\overline{Re}_o)^{0.53}\ (\overline{Pr}_{g,w})^{1/3} = (0.75)(225{,}000)^{0.53}\ (0.876) = 451$$

$$\bar{H}_{o,w} = \frac{\overline{Nu}_o\bar{k}_{g,w}}{\bar{D}_{h,o}} = \frac{(451)(0.82 \times 10^{-5})}{0.0834} = 0.04434 \text{ Btu/(sec)(sq ft)(}^{\circ}\text{F)}$$

$$\bar{H}_{o,w}\bar{l}_o = (0.04434)(0.262) = 0.01162$$

<u>Calculation of $\bar{H}_{i,w}$, $\bar{H}_f$, and $1 + \bar{\lambda}$.</u> - In the calculation of $\bar{H}_{i,w}$, the reference total temperature T'' equal to 900° R is employed because the velocity in the correlation equation (35) is based on the density evaluated at the total temperature. As before, the reference values of the blade dimensions are those at

$$y = 0.5$$

The value of $\bar{H}_i$ is computed based on $\bar{T}''$; $\bar{H}_{i,w}$ is determined from H_i by applying a temperature correction (appendix F). The value of $\bar{H}_f$ is then obtained from $\bar{H}_{i,w}$. The required computations are:

$$\overline{Re}_i = \frac{w}{Ag}\frac{\bar{D}_h}{\mu} = \frac{(0.01689)(0.00668)}{(0.00041)(32.17)(0.558\text{x}10^{-6})} = 15{,}330$$

$$\overline{Nu}_i = 0.019\ (\overline{Re}_i)^{0.8} = 0.019 \times (15{,}330)^{0.8} = 42.38$$

$$\bar{H}_i = \frac{\overline{Nu}_i\bar{k}}{\bar{D}_h} = \frac{(42.38)(0.657 \times 10^{-5})}{0.00668} = 0.04168 \text{ Btu/(sec)(sq ft)(}^{\circ}\text{F)}$$

$$\bar{H}_{i,w} = \bar{H}_i\left(\frac{\bar{T}''}{\bar{T}_B}\right)^{0.56} = 0.04168\left(\frac{900}{1300}\right)^{0.56} = 0.03393 \text{ Btu/(sec)(sq ft)(}^{\circ}\text{F)}$$

$$\varphi = \sqrt{\frac{2\ \bar{H}_{i,w}}{k_B\ \tau}} = \sqrt{\frac{(2)(0.03393)}{(0.004168)(0.0025)}} = 80.70$$

$$\varphi\ \bar{L}_f = (80.7)(0.0055) = 0.4439$$

$$\bar{H}_f = \frac{\bar{H}_{i,w}}{m + \tau}\left(\frac{2\ \tanh \varphi\ \bar{L}_f}{\varphi} + m\right)$$

$$= \frac{0.03393}{0.0050 + 0.0025}\left(\frac{2\ \tanh\ 0.4439}{80.70} + 0.0050\right) = 0.06935 \text{ Btu/(sec)(sq ft)(}^{\circ}\text{F)}$$

$$\bar{H}_f\bar{l}_i = (0.06935)(0.1295) = 0.008981$$

$$\bar{\lambda} = \frac{\bar{H}_{o,w}\ \bar{l}_o}{\bar{H}_f\ \bar{l}_i} = \frac{0.01162}{0.008981} = 1.294$$

Calculation of Cooling-Air Mach Number at Blade Tip

The Mach number of the cooling air at the blade tip is determined from equation (18).

$$I_c = \frac{w^2 R\, T''_T}{A_T{}^2 g p_T{}^2} = \frac{(0.01689)^2\ (53.3)(1160)}{(0.000194)^2\ (32.17)(5000)^2} = 0.583$$

The value of $\gamma = 1.370$, corresponding to $T''_T = 1160^\circ$ R, is obtained from figure 5. The value of $M_T = 0.629$ corresponds to $\gamma = 1.37$ and $I_c = 0.583$ in table I.

Calculation of Cooling-Air Total-Temperature

and Mach Number Distributions

The calculation schedules for $H_{o,w}l_o$ and $1 + \lambda$, T'' and dT''/dy, and M^2 and dM^2/dy are presented in this section. In the preparation of these schedules, the interval in y must be so chosen that the differences in the tabular values of dM^2/dy, dT''/dy, and M do not change too rapidly. (The differences in the tabular values of dM^2/dy normally change most rapidly.) In the present numerical example, the interval in y was chosen as 0.01 in the range of values of y from 0 to 0.15, 0.05 in the range of y from 0.15 to 0.40, and 0.1 in the range of y from 0.4 to 1.0. The size of the interval depends on the number of significant figures desired. No general rule can be given.

Numerical calculation of $H_{o,w}l_o$ and $1 + \lambda$. - Columns 2 to 11 in the following table constitute the computational schedule for determining $H_{o,w}l_o$ by equation (34). The quantity $1 + \lambda$ in column 3 must be assumed at first. If column 26 (in a subsequent part of the table) is widely different, the calculation must be repeated with a revised value of $1 + \lambda$ in column 3. The table shows the final calculation only.

1	2	3	4	5	6	7	8	9	10	11
	(1) and fig. 9		((3)-1) x (2)		(4)+(5)	(6)/(3)	1300/(7)	(8)$^{0.201}$		0.01162 x (9) (10)
y	$T_{g,e}$	$1+\lambda$	$\lambda T_{g,e}$	T_e	$\lambda T_{g,e} + T_e$	T_B	$\frac{\bar{T}_B}{T_B}$	$\left(\frac{\bar{T}_B}{T_B}\right)^{0.201}$	G_o	$H_{o,w} l_o$
0	1910	1.708	1352	1152.2	2504	1466	0.887	0.976	0.754	0.00855
.05	1965	1.762	1497	1139.3	2636	1496	.869	.972	.776	.00876
.10	2020	1.825	1667	1123.1	2790	1529	.850	.968	.796	.00895
.15	2073	1.922	1911	1107.0	3018	1570	.828	.963	.821	.00919
.20	2117	2.001	2119	1089.6	3209	1604	.810	.959	.843	.00939
.25	2135	2.080	2306	1071.0	3377	1624	.800	.956	.869	.00965
.30	2124	2.141	2423	1052.3	3475	1623	.801	.956	.893	.00992
.35	2103	2.203	2530	1033.6	3564	1618	.803	.957	.917	.01020
.40	2080	2.273	2648	1014.8	3663	1612	.806	.958	.943	.01050
.50	2029	2.325	2688	977.1	3665	1576	.825	.962	.994	.01111
.60	1971	2.442	2842	939.3	3781	1548	.840	.966	1.047	.01175
.70	1911	2.484	2836	900.5	3737	1504	.864	.971	1.104	.01246
.80	1847	2.534	2833	860.6	3694	1458	.892	.977	1.158	.01315
.90	1781	2.552	2764	820.6	3585	1405	.925	.985	1.218	.01394
1.00	1711	2.560	2669	779.7	3449	1347	.965	.993	1.278	.01475

Columns 12 through 25 in the following table constitute the computational schedule for determining $H_f l_i$ according to equations (37) and (38). In order to initiate the calculation of $1 + \lambda$, a trial value of λ at station 0 was estimated as follows:

$$\lambda_0 \approx \frac{\bar{\lambda}}{G_{f,0}} = \frac{1.317}{1.775} = 0.74$$

The first line $(y = 0)$ was then completed with the aid of columns 27 to 45 (in a subsequent part of the table) and the estimated value of λ was checked. If a value of λ differing substantially from $\lambda = 0.74$ had been obtained, the computations of the first line in the table would have been repeated. One repetition is usually sufficient. The trial value for λ in the second line $(y = 0.05)$ is estimated as follows:

$$\lambda_1 = \lambda_0 \frac{G_{f,0}}{G_{f,1}}$$

Trial values of λ for the third and the following lines may be estimated by noting the trend λ takes.

The values of T_e (column 5) are obtained by subtracting the value

$$(1-\Lambda)\ T''I_E$$

(column 30) from T'' (column 44) as in equation (8). The pertinent values of $I_{T,e}$ were found in table I. The operations to be performed in the remainder of the computational schedule may be determined from the column headings.

1	12	13	14	15	16	17	18	19
		$(12)^{0.35}$	$(8)^{0.28}$		0.4439×(13)(14)(15)	tanh (16)	$\frac{(16)}{L_f}$	$\frac{2\times(17)}{(18)}$
y	$\left(\frac{T''}{900^\circ}\right)$	$\left(\frac{T''}{\bar{T}''}\right)^{0.35}$	$\left(\frac{\bar{T}_B}{T_B}\right)^{0.28}$	G_1	φL_f	$\tanh \varphi L_f$	φ	$\frac{2 \tanh \varphi L_f}{\varphi}$
0	1.289	1.093	0.967	0.759	0.3561	0.3418	120.3	0.00568
.05	1.271	1.088	.962	.786	.3652	.3498	113.4	.00617
.10	1.252	1.082	.956	.811	.3724	.3561	107.3	.00664
.15	1.233	1.076	.949	.834	.3780	.3610	101.3	.00713
.20	1.213	1.070	.943	.857	.3838	.3660	96.7	.00757
.25	1.192	1.063	.939	.879	.3895	.3709	92.3	.00804
.30	1.171	1.057	.940	.900	.3969	.3773	89.2	.00846
.35	1.150	1.050	.940	.922	.4040	.3834	86.1	.00891
.40	1.129	1.043	.941	.939	.4091	.3877	83.3	.00931
.50	1.087	1.030	.948	.973	.4217	.3984	78.8	.01011
.60	1.044	1.015	.952	1.008	.4324	.4073	74.8	.01089
.70	1.001	1.000	.960	1.035	.4410	.4145	71.5	.01159
.80	.957	.985	.968	1.057	.4474	.4198	68.6	.01224
.90	.912	.968	.978	1.082	.4547	.4258	66.5	.01281
1.00	.867	.951	.990	1.096	.4580	.4285	64.8	.01323

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1	20	21	22	23	24	25	26
y	19 + 0.0050	$\frac{20}{0.01533}$	8 0.56 $\left(\frac{\overline{T}_B}{T_B}\right)^{0.56}$	12 0.7 $\left(\frac{T''}{900}\right)^{0.7}$	Figs. 3(a) and 3(c) G_f	0.008981× 21 22 23 24 $H_f l_i$	1 + $\frac{11}{25}$ 1 + λ
0	0.01068	0.697	0.935	1.194	1.781	0.01245	1.687
.05	.01117	.729	.924	1.183	1.613	.01154	1.759
.10	.01164	.759	.913	1.170	1.480	.01078	1.830
.15	.01213	.791	.900	1.158	1.365	.01011	1.909
.20	.01257	.820	.889	1.145	1.284	.00963	1.975
.25	.01304	.851	.883	1.131	1.207	.00921	2.048
.30	.01346	.878	.883	1.117	1.146	.00891	2.113
.35	.01391	.909	.884	1.103	1.100	.00874	2.167
.40	.01431	.933	.886	1.089	1.059	.00856	2.227
.50	.01511	.986	.898	1.060	1.003	.00845	2.315
.60	.01589	1.037	.907	1.031	.967	.00842	2.395
.70	.01659	1.082	.921	1.001	.953	.00854	2.459
.80	.01724	1.125	.938	.970	.946	.00870	2.511
.90	.01780	1.162	.957	.938	.965	.00904	2.542
1.00	.01823	1.189	.980	.905	.988	.00936	2.576

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<u>Solution for total-temperature distribution of cooling air</u>. - The computational schedule for solving equation (8) for dT''/dy is given in columns 27 to 35.

1	27	28	29	30	31	32	33
	1-(1)	Fig. 4	$\frac{0.10 \frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}$	(29) T"	(2)+(30)-(44)	$\frac{17.76 (11)(31)}{(28)(26)}$	$\frac{2.278 \times (27)}{(28)}$
y	1-y	c_p		$T''-T_e$	$T_{g,e}-T_e$		
0	1.00	0.2535	0.00678	7.9	758	269.1	9.0
.05	.95	.2530	.00463	5.3	826	288.8	8.6
.10	.90	.2524	.00350	3.9	897	308.7	8.1
.15	.85	.2520	.00274	3.0	967	328.1	7.7
.20	.80	.2511	.00223	2.4	1029	346.0	7.3
.25	.75	.2506	.00185	2.0	1067	356.3	6.8
.30	.70	.2501	.00157	1.7	1076	358.7	6.4
.35	.65	.2495	.00134	1.4	1074	359.8	5.9
.40	.60	.2490	.00116	1.2	1071	360.2	5.5
.50	.50	.2478	.00089	.9	1060	364.6	4.6
.60	.40	.2470	.00071	.7	1042	367.6	3.7
.70	.30	.2461	.00058	.5	1023	369.3	2.8
.80	.20	.2452	.00049	.4	999	378.9	1.9
.90	.10	.2443	.00043	.4	974	388.3	.9
1.00	0	.2435	.00038	.3	946	395.1	0

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Columns 35 to 38 constitute a difference table for dT''/dy. An equation similar to equation (24), but written for T'', is solved for T''_{n+1} in columns 39 to 44.

1	34	35	36	37	38	39	40
	$\frac{8.48}{(28)}$	(32) + (33) + (34)				$-\frac{(36)}{2}$	$-\frac{(37)}{12}$
y		$-\frac{dT''}{dy}$	$\Delta_1\left(-\frac{dT''}{dy}\right)$	$\Delta_2\left(-\frac{dT''}{dy}\right)$	$\Delta_3\left(-\frac{dT''}{dy}\right)$	$-\frac{\Delta_1}{2}$	$-\frac{\Delta_2}{12}$
0	33.5	312					
.05	33.5	331	19			-10	
.10	33.6	350	19	0		-10	0
.15	33.7	370	20	+1	+1	-10	0
.20	33.8	387	17	-3	-4	-9	0
.25	33.8	397	10	-7	-4	-5	1
.30	33.9	399	2	-8	-1	-1	1
.35	34.0	400	1	-1	+7	0	0
.40	34.1	400	0	-1	0	0	0
.50	34.2	403	3	3	+4	-2	0
.60	34.3	406	3	0	-3	-1	0
.70	34.5	407	1	-2	-2	-1	0
.80	34.6	415	8	7	+9	-4	-1
.90	34.7	424	9	1	-6	-5	0
1.00	34.8	430	6	-3	-4	-3	0

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1	41	42	43	44	45
y	$-\frac{38}{24}$ $-\frac{\Delta_3}{24}$	35 + 39 + 40 + 41	$(y_{n+1} - y_n)$ 42 $-\Delta T''$	T''	$-\frac{1}{T''}\frac{dT''}{dy}$
0				1160	0.269
.05		321	16	1144	.289
.10		340	17	1127	.311
.15	0	360	18	1109	.334
.20	0	378	19	1090	.355
.25	0	392	20	1070	.371
.30	0	398	20	1050	.380
.35	0	400	20	1030	.388
.40	0	400	20	1010	.396
.50	0	401	40	970	.415
.60	0	405	40	930	.437
.70	0	406	41	889	.458
.80	0	411	41	848	.489
.90	0	419	42	806	.526
1.00	0	427	43	763	.564

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Solution for Mach number distribution. - Columns 46 to 59 constitute the solution of equation (9) for dM^2/dy.

1	46	47	48	49	50	51	52	53
y	1 + 0.2686× (27)	Table I I_R	247.7× (46)(47) / (44)	(1) and fig. 4 D_h	0.00780 / (49) $\frac{4fb}{D_h}$	(50) - (48)	(1) and fig. 7 $\frac{1}{A}\frac{dA}{dy}$	Table I I_T
0	1.2686	1.9799	0.536	0.00440	1.773	1.237	2.20	1.0834
.01	1.2659	2.1626	.586	.00445	1.753	1.167	2.174	.89586
.02	1.2632	2.3428	.635	.00450	1.733	1.098	2.130	.76158
.03	1.2605	2.5220	.684	.00455	1.714	1.030	2.080	.66047
.04	1.2579	2.7045	.735	.00460	1.696	.961	2.030	.58025
.05	1.2552	2.8872	.785	.00465	1.677	.892	2.000	.51628
.06	1.2525	3.0665	.834	.00470	1.660	.826	1.955	.46516
.07	1.2498	3.2630	.888	.00475	1.642	.754	1.920	.41886
.08	1.2471	3.4536	.941	.00480	1.625	.684	1.880	.38191
.09	1.2444	3.6454	.994	.00485	1.608	.614	1.848	.35029
.10	1.2417	3.8545	1.052	.00490	1.592	.540	1.820	.32107
.11	1.2391	4.0462	1.105	.00495	1.576	.471	1.786	.29805
.12	1.2364	4.2534	1.163	.00500	1.560	.397	1.750	.27652
.13	1.2337	4.4563	1.219	.00505	1.545	.326	1.718	.25812
.14	1.2310	4.6745	1.280	.00510	1.529	.249	1.692	.24079
.15	1.2283	4.8852	1.340	.00515	1.515	.175	1.661	.22603
.20	1.2149	6.0261	1.664	.00540	1.444	- .220	1.531	.16923
.25	1.2015	7.2632	2.020	.00564	1.383	- .637	1.425	.13255
.30	1.1880	8.6378	2.421	.00587	1.329	- 1.092	1.325	.10663
.35	1.1746	10.075	2.846	.00610	1.279	- 1.567	1.240	.08842
.40	1.1612	11.629	3.312	.00631	1.236	- 2.076	1.170	.07458
.50	1.1343	15.081	4.368	.00668	1.168	- 3.200	1.050	.05528
.60	1.1074	18.945	5.588	.00695	1.122	- 4.466	.940	.04283
.70	1.0806	23.173	6.977	.00715	1.091	- 5.886	.809	.03434
.80	1.0537	27.948	8.602	.00730	1.068	- 7.534	.660	.02806
.90	1.0269	32.155	10.148	.00741	1.053	- 9.095	.490	.02417
1.00	1.0000	35.336	11.471	.00749	1.041	-10.430	.254	.02186

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1	54	55	56	57	58	59	60
y	Table I $-I_f$	Table I I_A	53 × 45	54 × 51	55 × 52	56 + 57 + 58 $\frac{dM^2}{dy}$	$\Delta_1 \frac{dM^2}{dy}$
0	-0.3808	-1.4744	-0.2914	-0.471	-3.244	-4.006	
.01	- .29590	-1.1999	- .2446	- .345	-2.609	-3.199	0.807
.02	- .23743	-1.0483	- .2110	- .261	-2.233	-2.705	.494
.03	- .19501	- .93091	- .1856	- .201	-1.936	-2.323	.382
.04	- .16257	- .83537	- .1654	- .156	-1.696	-2.017	.306
.05	- .13760	- .75735	- .1492	- .123	-1.515	-1.787	.230
.06	- .11832	- .70133	- .1365	- .098	-1.371	-1.606	.181
.07	- .10145	- .63482	- .1247	- .076	-1.219	-1.420	.186
.08	- .08844	- .58694	- .1154	- .060	-1.103	-1.278	.142
.09	- .07766	- .54525	- .1074	- .048	-1.008	-1.163	.115
.10	- .06803	- .50606	- .0999	- .037	- .921	-1.058	.105
.11	- .06069	- .47473	- .0941	- .029	- .848	- .971	.087
.12	- .05403	- .44498	- .0885	- .021	- .779	- .889	.082
.13	- .04851	- .41922	- .0838	- .016	- .720	- .820	.069
.14	- .04347	- .39463	- .0793	- .011	- .668	- .758	.062
.15	- .03931	- .37345	- .0755	- .007	- .620	- .703	.055
.20	- .02456	- .28934	- .0601	.005	- .443	- .498	.205
.25	- .01631	- .23249	- .0492	.010	- .331	- .370	.128
.30	- .01122	- .19081	- .0405	.012	- .253	- .282	.088
.35	- .008084	- .16069	- .0343	.013	- .199	- .220	.062
.40	- .005969	- .13721	- .0295	.012	- .161	- .179	.041
.50	- .003467	- .10363	- .0229	.011	- .109	- .121	.058
.60	- .002163	- .081334	- .0187	.010	- .076	- .085	.036
.70	- .001429	- .065825	- .0157	.008	- .053	- .061	.024
.80	- .0009737	- .054164	- .0137	.007	- .036	- .043	.018
.90	- .0007321	- .046866	- .0127	.007	- .023	- .029	.014
1.00	- .0006038	- .042512	- .0123	.006	- .011	- .017	.012

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Columns 59 to 62 constitute a difference table. Equation (24) is solved for M^2_{n+1} in columns 63 to 68; the column headings indicate the required calculations.

1	61	62	63	64	65	66	67	68	69	70
			$-\frac{60}{2}$	$-\frac{61}{12}$	$-\frac{62}{24}$	59+63+64+65	$(y_{n-1}-y_n)$ ×66		Computed	Trial
y	$\Delta_2 \frac{dM^2}{dy}$	$\Delta_3 \frac{dM^2}{dy}$	$-\frac{\Delta_1}{2}$	$-\frac{\Delta_2}{12}$	$-\frac{\Delta_3}{24}$		AM^2	M^2	M	M
0								0.3956	0.629	0.629
.01			-0.404			-3.603	-0.03603	.3596	.600	.600
						-3.535	- .03567	.3599		
.02	-0.313		- .247	0.026		-2.978	- .02978	.3298	.574	.575
						-2.936	- .02933	.3306		
.03	- .112	0.201	- .191	.009	-0.008	-2.513	- .02513	.3048	.552	.553
						-2.507	- .02512	.3055		
.04	- .076	.036	- .153	.006	- .002	-2.166	- .02166	.2838	.533	.533
.05	- .076	- .000	- .115	.006	0	-1.896	- .01896	.2648	.515	.515
.06	- .049	- .027	- .091	.004	.001	-1.692	- .01692	.2499	.498	.498
.07	.005	.054	- .093	0	- .002	-1.515	- .01515	.2328	.482	.483
.08	- .044	- .049	- .071	.004	.002	-1.343	- .01343	.2194	.468	.469
.09	- .027	.017	- .058	.002	- .001	-1.220	- .01220	.2072	.455	.456
.10	- .010	.017	- .053	.001	- .001	-1.111	- .01111	.1961	.443	.443
.11	- .018	- .008	- .044	.002	0	-1.013	- .01013	.1860	.431	.432
.12	- .005	.013	- .042	0	- .001	- .930	- .00930	.1767	.420	.421
.13	- .013	- .008	- .035	.001	0	- .854	- .00854	.1682	.410	.411
.14	- .007	.006	- .031	.001	0	- .788	- .00788	.1603	.400	.401
.15	- .007	.000	- .028	0	0	- .730	- .00730	.1530	.391	.392
.20	- .150	.224	- .103	.013	- .009	- .597	- .02985	.1232	.351	.352
.25	- .077	.073	- .064	.006	- .003	- .431	- .02155	.1017	.319	.320
.30	- .040	.037	- .044	.003	- .002	- .325	- .01625	.0855	.292	.293
.35	- .026	.014	- .031	.002	- .001	- .250	- .01250	.0730	.272	.271
.40	- .021	.005	- .021	.002	0	- .198	- .00990	.0631	.252	.252
.50	- .045	.068	- .029	.004	- .003	- .149	- .01490	.0482	.220	.221
.60	- .022	.023	- .018	.002	- .001	- .102	- .0102	.0380	.195	.197
.70	- .012	.010	- .012	.001	0	- .072	- .0072	.0308	.175	.178
.80	- .006	.006	- .009	.000	0	- .050	- .0050	.0258	.161	.162
.90	- .004	.002	- .007	0	0	- .036	- .0036	.0222	.149	.151
1.00	- .002	.002	- .006	0	0	- .023	- .0023	.0199	.141	.144

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When the differential equations for the flow in the blade coolant passages have been solved for the total temperature and the Mach number distributions, the pressures and the velocities can readily be found from equations (16) and (17), respectively. The computational schedule for obtaining the static-pressure and velocity distributions of the cooling air along the blade coolant passage is as follows:

1	71	72	73	74	75
y	From (69) M	From (44) T"	$1 + 0.185 (71)^2$ $1 + \frac{\gamma-1}{2} M^2$	From fig. 3 A	$\frac{0.000194}{(74)}$ $\frac{A_T}{A}$
0	0.629	1160	1.0732	0.000194	1.000
.05	.515	1144	1.0491	.000215	.9023
.10	.443	1127	1.0363	.000236	.8220
.15	.391	1109	1.0284	.000259	.7490
.20	.351	1090	1.0229	.000281	.6904
.25	.319	1070	1.0189	.000303	.6403
.30	.292	1050	1.0159	.000324	.5988
.35	.272	1030	1.0137	.000346	.5607
.40	.252	1010	1.0118	.000367	.5286
.50	.220	970	1.0090	.000409	.4743
.60	.195	930	1.0072	.000451	.4302
.70	.175	889	1.0059	.000491	.3951
.80	.161	848	1.0049	.000530	.3660
.90	.149	806	1.0042	.000562	.3452
1.00	.141	763	1.0038	.000590	.3288

1	76	77	78	79	80	81
	$\frac{0.629}{(71)}$	$\sqrt{\frac{(72)}{1160}}$	$\frac{1.0732}{(73)}$	$\sqrt{(78)}$	(75)(76)(77)(79)	$\frac{(77)(79)}{(76)}$
y	M_T/M	$\sqrt{T''/T''_T}$	$\frac{1 + \frac{\gamma-1}{2} M_T^2}{1 + \frac{\gamma-1}{2} M^2}$		p/p_T	W/W_T
0	1.000	1.0000	1.000	1.000	1.000	1.000
.05	1.221	.9931	1.023	1.011	1.106	.8223
.10	1.420	.9856	1.036	1.018	1.171	.7066
.15	1.609	.9778	1.044	1.022	1.204	.6211
.20	1.792	.9694	1.049	1.024	1.228	.5539
.25	1.972	.9604	1.053	1.026	1.244	.4997
.30	2.154	.9514	1.057	1.028	1.261	.4541
.35	2.313	.9422	1.059	1.029	1.257	.4192
.40	2.496	.9331	1.061	1.030	1.268	.3851
.50	2.859	.9144	1.064	1.032	1.280	.3301
.60	3.226	.8954	1.066	1.032	1.282	.2864
.70	3.594	.8754	1.067	1.033	1.284	.2516
.80	3.907	.8550	1.068	1.033	1.263	.2261
.90	4.221	.8335	1.069	1.034	1.256	.2042
1.00	4.461	.8110	1.069	1.034	1.230	.1880

The variations in static pressure, total temperature, Mach number, and velocity of the coolant from the tip to the root of the blade coolant passage, as obtained in the numerical example, are presented in figure 7.

APPROXIMATE CLOSED-FORM SOLUTION OF ENERGY EQUATION

In the preceding schedule of computations, 45 columns are used to calculate the temperature distributions and 25 to compute the Mach numbers along the passage. Of the first 45 columns, 38 could be eliminated if average values of the coefficients in equation (8) could be used so as to make possible a closed-form solution. This simplification will accordingly be made and the results will be compared with those just obtained.

Inspection shows that the energy equation (8) can be reduced to the form of a linear equation of the first degree with constant coefficients by the following simplifications:

1. Neglect the term $(1-\Lambda)\ I_E$, which is usually less than 2 percent of unity.

2. Use mean constant values for the coefficients of $T_{g,e}$, T'' and $1 - y$, and for the last term.

Under these conditions the solution of equation (8) is (appendix B)

$$T'' = K\ e^{K_1 y} + K_2 y + K_3 - K_1\ e^{K_1 y} \int_0^y T_{g,e}\ e^{-K_1 y}\ dy \qquad (39)$$

where

$$K_1 = \frac{b}{w} \frac{H_{o,w} l_o}{\bar{c}_p\ (1+\bar{\lambda})}$$

$$K_2 = -\frac{b\omega^2\ w\ (1+\bar{\lambda})}{Jg\bar{H}_{o,w}\bar{l}_o}$$

$$K_3 = \frac{\omega^2\ w\ (1+\bar{\lambda})}{Jg\bar{H}_{o,w}\bar{l}_o}\ (b+r_h) - \frac{\bar{c}_p \omega^2 w^2 (1+\bar{\lambda})^2}{Jg\bar{H}_{o,w}{}^2 \bar{l}_o{}^2}$$

and where K is the integration constant.

If $T_{g,e}$ in equation (39) can also be replaced by a constant mean value, the last term simplifies to $\bar{T}_{g,e}$ so that equation (39) can be written (appendix B)

$$T'' = Ke^{K_1 y} + K_2 y + K_3 + \bar{T}_{g,e} \qquad (40)$$

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If the total temperature of the cooling air at the tip is known, the value of the constant of integration is determined by making the substitutions

$$T'' = T''_T$$

and

$$y = 0$$

in equation (40). Thus,

$$K = T''_T - K_3 - \overline{T}_{g,e} \tag{41}$$

A numerical example employing equation (40) is presented in appendix G. A method of calculating $\overline{T}_{g,e}$ from the temperature at the combustion-chamber exit is given in appendix H.

The approximate cooling-air total-temperature distribution may be determined independently of the momentum equation by means of assumptions 1 and 2. (See appendix I.) These assumptions lead to equation (39). The additional assumption that $\overline{T}_{g,e}$ is constant leads to equation (40). In addition, the constant K may be evaluated at either the root or the tip of the blade coolant passage (appendix B), depending on the location of the known conditions.

COMPARISON OF SOLUTIONS AND DETERMINATION

OF BLADE-METAL TEMPERATURES

Three solutions have been presented for the energy equation (8), whereas only an open-form solution has been presented for the momentum equation (9). A discussion of the three solutions for the energy equation as applied to the numerical example follows:

<u>Solution A</u>. - The energy equation (8) was solved numerically and simultaneously with the momentum equation to obtain the distribution of T'' along the blade coolant passage. When such a

solution was employed in the numerical example, the following parameters, which depend on the blade geometry, were allowed to vary with y: A, D_h, l, and L_f, as specified by figures 3(a), 3(b), 3(c), and 3(d), respectively. In addition, the distributions of W/W_g and $T_{g,e}$ specified in figures 3(f) and 3(g) were assumed. The variation of c_p with temperature was also taken into account. 1199

Solution B. - A solution for the energy equation in which all the parameters dependent on the geometry of the blade were assumed constant and equal to the respective values at midspan $(y = 0.5)$ is presented in equation (39). An average value of W/W_g was employed and the heat-transfer coefficients were evaluated at an assumed average blade-metal temperature, which was later checked. The variation in $T_{g,e}$ along the span as specified in figure 3(g) was used

Solution C. - The energy equation may be solved with equation (40) by assuming all parameters, including $T_{g,e}$, constant and equal to their respective mean values. For the purpose of comparing results with the numerical example of this report, the integrated mean value $T_{g,e} = 1967^{\circ}$ R given in figure 3(g) was used.

Comparison of Three Methods of Solution

Solutions of the energy equation in the three forms (equations (8), (39), and (40)) were calculated by using the values of the parameters listed for the numerical example. Errors in the calculation of the cooling-air total temperature obtained by comparing solutions A, B, and C are presented in figure 8(a). The three distributions agree reasonably well, although average values of certain parameters were employed in solutions B and C. The maximum deviations from the values of T'' obtained from solution A are as follows:

Solution	y	Deviation of T'' (percent)
B	0.8	1.6
C	1.0	2.3

For solution C, the values of T'' agree closely with the values obtained from solution A except near the root of the blade. Inasmuch as T'' affects the values of M, p, and W, further comparisons will be made employing solution C, in which the largest deviation in T'' occurred.

Variations in M, p, T'', and W obtained from solution A and equations (9), (16), and (17) are shown in figure 7. Similar computations were made for solution C; these computations agreed quite closely with the values in figure 7. Figure 8 shows the percentage errors due to the use of the simplified energy equation in solution C. The maximum percentage deviations in two ratios between solution A and solution C are

Ratio	y	Deviation (percent)
p/p_T	0.5	2.1
M/M_T	.5	1.5

The maximum errors are thus approximately 2 percent.

Blade-Metal Temperature Distribution

The blade-metal temperature distribution obtained from solution A is presented in figure 9. For comparison, plots of $T_{g,e}$ and T'' are shown in the same figure. No attempt was made to account for the effect on T_B of heat conduction to the rim, but the region of appreciable influence is indicated. Also shown in figure 9 are two approximate distributions of the blade temperature obtained from the equation

$$T_B = \frac{\lambda T_{g,e} + T''}{1 + \lambda} \tag{42}$$

In one of these plots of T_B, a constant mean value of $\bar{\lambda}$ was used. In the second of these plots, λ was considered to vary only with the change in geometric configuration of the blade along the span and is given by the following equation:

$$\lambda = \frac{\bar{\lambda}}{G_f} \tag{43}$$

In both cases, the values of T'' obtained from solution C and the values of $T_{g,e}$ shown in figure 9 were used.

In the case of a constant mean value for λ, the agreement in T_B with solution A is rather poor near the tip of the blade. At the tip of the blade, the constant value of λ caused a deviation in T_B of 9 percent from the value obtained from solution A; the maximum value of T_B deviated from the maximum value of T_B from solution A by 4 percent.

In the case of a variable λ, rather close agreement was obtained between the approximate value of T_B and the value obtained in solution A. At the maximum values of T_B, the deviation for a variable λ is 1 percent.

Conclusions Based on Numerical Example

The numerical example presented suggests the following conclusions:

1. Solution C may be employed with reasonable accuracy in the computation of the distributions of T'', M, W, and p in the blade coolant passage.

2. Solution A should be employed for a close distribution of T_B along the blade coolant passage.

3. In order to estimate values of T_B, solution C may be employed in conjunction with a value of λ that is allowed to vary according to the variation in blade geometry.

The numerical example chosen is rather an extreme case in that the flow-area variation was about 300 percent along the coolant passage; this value is larger than likely for a practical cooled blade. In most cases, the variations along the span would be less than for this example and the differences between the approximate curves (obtained by using constant mean values) and the more exact curves (obtained by using values that are allowed to vary along the span) would therefore be smaller.

Effect of Errors in Friction and Heat-Transfer Coefficients

The use of mean constant values for the coefficients of the variables T and y in the working equation (8) has previously been shown to produce only small variations from the values of the pressure and velocity ratios obtained by taking all variations along the span into account.

The effect of changes in the values of these constants on the computed pressure ratio has been studied by means of additional unpublished calculations for constant flow area; the changes due to the other factors could thus be evaluated.

Under these conditions, the influence of the friction coefficient (appendix J) was found to depend markedly on the Mach number at the tip. For example, in one case for which the exit Mach number was approximately 0.4, a change of 30 percent in the friction coefficient produced a change of only 8.0 percent in the pressure ratio p_h/p_T. At an exit Mach number of approximately 0.8, an 8-percent change in the friction coefficient produced a change in the pressure ratio of about 17 percent. The pressure ratio is thus about six times as sensitive to variations in the friction coefficient at a Mach number of 0.8 as at a Mach number of 0.4.

The effects of variation in Mach number on the heat-transfer coefficient are the reverse of those noted for the friction coefficient; that is, at the low Mach numbers, changes in the heat-transfer coefficient have a greater relative influence on the pressure ratio than at the high Mach numbers. Because changes in the heat-transfer rate produce changes in both the energy and momentum equations, the heat-transfer rate affects the temperature distribution.

The equations were recomputed for an exit Mach number of 0.4 with the inside heat-transfer coefficient changed 10 percent. A change in pressure ratio of 0.4 percent was obtained. The pressure ratio is thus fairly insensitive to changes in the heat-transfer coefficient. In a hollow blade without fins, variations in the heat-transfer coefficient would be slightly more influential; a 10-percent change in the heat-transfer coefficient would effect a change in the pressure ratio of 0.9 percent.

SUMMARY AND CONCLUSIONS

The effects of area change, wall friction, heat transfer, and rotation on the air flow in the coolant passage of a hollow turbine blade were investigated. Special forms of the differential equations for the conservation of energy and momentum were derived from the equation of state, the continuity equation, and the general forms of the energy and momentum equations. The differential energy equation was developed in terms of the coolant total temperature and the passage location; the differential momentum equation was developed in terms of the coolant Mach number and the passage location.

A numerical method was devised for solving these two differential equations simultaneously under the general conditions that all the coefficients in the equations could be arbitrarily varied along the coolant passage; the method also permits varying the combustion-gas effective temperature and relative velocity at the entrance to the rotor. Equations were derived for calculating the static pressure and the relative velocity of the coolant directly from the solutions of the differential equations.

A simplified solution of the energy equation was obtained in which mean constant values were used for the coefficients of the variables T and y so that the solution could be given in a simple closed form.

Tables of the pertinent Mach number functions previously published were extended so as to include all the required functions of Mach number, a smaller interval in Mach number, and values of the ratio of specific heats equal to 1.37 and 1.40.

Numerical examples were solved by use of both the general solution of the differential equations and a solution using the simplified solution of the energy equation. Plots of the results obtained for an example solved by the two methods indicate that the simplified solution results in distributions of the Mach number, static pressure, total temperature, velocity relative to the blade, and static pressure of the coolant that are within 2 percent of the values obtained from the more rigorous general solution.

The most precise values of the blade-metal temperature distribution were obtained from the general solution, which requires the simultaneous numerical solution of the differential equations for the conservation of energy and momentum. The blade-metal temperature distribution may be estimated by using the simplified solution of the energy equation. In this case, the ratio of the outside to the inside heat-transfer coefficient was varied according to the variations in the geometrical configuration only when used in the relation

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for the blade-metal temperature. The combustion-gas effective temperature distribution was also used in the relation for the blade-metal temperature.

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APPENDIX A

SYMBOLS

Blade sketches and configurations are shown in figure 1 with the pertinent symbols indicated.

When c_p, M, p, Re, T, γ, μ, and ρ appear without subscripts, the fluid inside the passage is referred to. The subscript g is employed to designate the combustion gas flowing outside the blade.

The following symbols are used in this report:

A	flow area, sq ft
A_B	cross-sectional area of blade, sq ft
a	velocity of sound, ft/sec
b	blade length or span, ft
c_p	specific heat at constant pressure, Btu/(lb)(°F)
D_h	hydraulic diameter, $\dfrac{\text{4 times flow area}}{\text{wetted perimeter}}$, ft
dp_{fr}	friction differential pressure drop, lb/sq ft
F_b	generalized body force, lb/(lb coolant)
f	friction coefficient
G	geometry factor
Gr	Grashof number, $\dfrac{b^3\rho^2\beta(\omega^2 r)(T_B-T_e)}{\mu^2}$
g	standard acceleration of gravity, ft/sec^2
H	convection heat-transfer coefficient, Btu/(sec)(sq ft)(°F)
I_A	influence coefficient for term involving rate of area change (equation (12))

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I_c	function of Mach number and ratio of specific heats (equation (18))
I_f	influence coefficient for term involving friction (equation (13))
I_R	influence coefficient for term involving rotation (equation (11))
I_T	influence coefficient for term involving rate of temperature change (equation (14))
$I_{T,e}$	influence coefficient for effective temperature correction (equation (10))
J	mechanical equivalent of heat, ft-lb/Btu
K,K_1,K_2,K_3	constants
k	thermal conductivity, Btu/(sec)(ft)(°F)
L_f	mean half-width of fins in blade coolant passage, ft (See fig. 1(d).)
l	perimeter, ft
M	Mach number relative to blade, W/a
m	distance between adjacent fins in blade coolant passage, ft (See fig. 1(d).)
N	rotative speed, rps
Nu	Nusselt number, HD_h/k
Pr	Prandtl number, $c_p\mu g/k$
p	static pressure, lb/sq ft absolute
p''	total pressure with respect to rotating passage, lb/sq ft
q	heat added to coolant, Btu/(lb coolant)
R	gas constant, ft-lb/(lb)(°R)

Re	Reynolds number, $\rho D_h W/\mu$
r	radius, ft
T	static temperature, $^{\circ}R$
T"	total temperature with respect to rotating passage, $^{\circ}R$
u	blade speed, ft/sec
V	absolute velocity, ft/sec
W	velocity relative to rotating passage, ft/sec
W_g	velocity of combustion gas relative to blade at entrance to rotating blades, ft/sec (See fig. 2(b).)
w	coolant weight flow per blade, lb/sec
x	fraction of outside blade perimeter having laminar boundary layer
y	nondimensional coordinate $(r_h+b-r)/b$
α	stator-blade exit angle measured from tangential direction, deg (See fig. 2(b).)
β	thermal coefficient of volume expansion at constant pressure, $^{\circ}R^{-1}$, $\rho \frac{d(1/\rho)}{dT}$
γ	ratio of specific heats
$\Delta_1,\Delta_2,\Delta_3$	first, second, and third differences when employed with a variable
$\Delta_{1,r},\Delta_{2,r},\Delta_{3,r}$	first, second, and third differences for reduced interval
ζ	heat-transfer parameter, $(r-r_h)\left(\frac{H_o l_o+H_i l_i}{k_B A_B}\right)$
η	stator efficiency

θ	temperature-ratio factor
Λ	recovery coefficient
λ	ratio, $(H_o l_o)/(H_i l_i)$
μ	viscosity, slugs/(ft)(sec)
ρ	mass density, slugs/cu ft
τ	thickness of fins in blade coolant passage, ft (See fig. 1(d).)
φ	heat-transfer parameter, $(2H_i)/(k_B\tau)$, ft^{-1}
ω	angular velocity, radians/sec

Subscripts:

av	average
B	blade (with T denotes average at radius in question)
e	effective
ex	stator exit
f	fins
g	combustion gas
h	entrance to blade coolant passage (sometimes blade root)
i	inside of blade
id	ideal
in	stator inlet
n	any number
o	outside of blade
r	reduced size

T	tip of blade
u	tangential
vK	von Kármán
w	gas properties based on wall temperatures
$0,\frac{1}{2},1,2, \ldots n$	stations in coolant passage

Reference or mean values of any quantity are designated by a bar.

APPENDIX B

DERIVATION OF ENERGY EQUATION

The general form of the energy equation is (reference 2)

$$dq + F_b \frac{dr}{J} = c_p\, dT + d\, \frac{W^2}{2gJ} \tag{3}$$

In a rotating passage, F_b is the centrifugal force on the fluid so that

$$F_b\, dr = \frac{\omega^2 r\, dr}{g} \tag{44}$$

The total temperature of the cooling air may be defined as

$$T'' = T + \frac{W^2}{2Jgc_p} \tag{6}$$

which when differentiated and multiplied by c_p yields

$$c_p\, dT'' = c_p\, dT + d\, \frac{W^2}{2gJ} \tag{45}$$

Now

$$dq = \frac{(T_B - T_e)\, H_i l_i\, dr}{w} \tag{46}$$

and the heat entering the outside blade surface from the hot gas equals the heat flowing into the cooling air through the inside blade surface so that

$$H_{o,w}l_o(T_{g,e} - T_B) = H_i l_i(T_B - T_e) \tag{47}$$

If λ is defined as

$$\lambda = \frac{H_{o,w}l_o}{H_i l_i} \tag{48}$$

equation (47) becomes

$$T_B = \frac{\lambda T_{g,e} + T_e}{1 + \lambda} \tag{47a}$$

The substitution of equation (47a) into equation (46) results in

$$dq = \frac{\lambda}{1 + \lambda} \frac{(T_{g,e} - T_e) H_i l_i \, dr}{w} \tag{49}$$

Substitution of equations (44), (45), (48), and (49) into equation (3) and division of the resulting equation by $c_p dr$ results in the expression

$$\frac{dT''}{dr} = \frac{\omega^2 r}{gJc_p} + \frac{H_{o,w}l_o}{(1 + \lambda)c_p w} (T_{g,e} - T_e) \tag{50}$$

The recovery coefficient is defined as

$$\Lambda_i = \frac{T_e - T}{T'' - T} \tag{51}$$

from which

$$T_e = \Lambda_i\, T'' + (1 - \Lambda_i)\, T \tag{52}$$

The temperature T in equation (52) may be replaced by its equivalent

$$T = \frac{T''}{1 + \frac{\gamma-1}{2} M^2} \tag{53}$$

so that after algebraic manipulation equation (52) becomes

$$T_e = T''\left(\frac{1 + \Lambda_i \frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}\right) \tag{54}$$

Addition and subtraction of $(1 - \Lambda_i)\frac{\gamma-1}{2} M^2$ to the numerator of equation (54) results in the following relation between T_e and T'':

$$T_e = T''\left[1 - (1-\Lambda_i)\,\frac{\frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2}\right] \tag{55}$$

The replacement of

$$I_{T,e} = \frac{\frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2} \tag{56}$$

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in equation (55) and the substitution of the same equation into equation (51) results in the following form of the energy equation:

$$\frac{dT''}{dr} = \frac{\omega^2 r}{gJc_p} + \frac{H_{o,w} l_o}{(1+\lambda)c_p w}\left\{T_{g,e} - T''\left[1 - (1 - \Lambda_1)\, I_{T,e}\right]\right\} \tag{57}$$

It is convenient to replace r by the nondimensional quantity y defined as

$$r = r_h + (1 - y)\, b \tag{58}$$

from which

$$dr = -b\, dy \tag{59}$$

The introduction of equations (58) and (59) into equation (57) results in the working form of the energy equation

$$\frac{dT''}{dy} = -\frac{bH_{o,w} l_o}{wc_p\,(1+\lambda)}\left\{T_{g,e} - T''\left[1 - (1 - \Lambda_1)\, I_{T,e}\right]\right\} - \frac{b^2\omega^2}{Jgc_p}(1-y) - \frac{br_h\omega^2}{Jgc_p} \tag{8}$$

<u>Closed-form solution of energy equation with $T_{g,e}$ as function of y.</u> - In many cases, Λ_1 can be assumed equal to unity in equation (8) without introducing a sizeable error in the resulting T'' distribution. If mean values are then chosen for the coefficients, equation (8) may be written in the following form:

$$\frac{dT''}{dy} - \frac{b\overline{H}_{o,w}\overline{l}_o}{w\overline{c}_p(1+\overline{\lambda})}T'' = \frac{b^2\omega^2}{Jg\overline{c}_p}y - \frac{b^2\omega^2}{Jg\overline{c}_p} - \frac{b\overline{H}_{o,w}\overline{l}_o}{w\overline{c}_p(1+\overline{\lambda})}T_{g,e} - \frac{br_h\omega^2}{Jg\overline{c}_p} \tag{60}$$

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where $T_{g,e}$ is a known function of y. Equation (60) is a first-order linear differential equation with constant coefficients and its solution is given by (reference 17, p. 50)

$$T'' \, e^{\int - \frac{b\bar{H}_{o,w}\bar{l}_o dy}{w\bar{c}_p(1+\bar{\lambda})}} = \int e^{\int - \frac{b\bar{H}_{o,w}\bar{l}_o dy}{w\bar{c}_p(1+\bar{\lambda})}} \int \left[\frac{b^2\omega^2}{Jg\bar{c}_p} y - \frac{b^2\omega^2}{Jg\bar{c}_p} - \frac{br_h\omega^2}{Jg\bar{c}_p} - \frac{b\bar{H}_{o,w}\bar{l}_o}{w\bar{c}_p(1+\bar{\lambda})} T_{g,e}\right] dy + K \tag{61}$$

where K is the constant of integration. The integration of equation (61) results in

$$T'' = K\,e^{K_1 y} + K_2\,y + K_3 - K_1\,e^{K_1 y}\int_0^y T_{g,e}\,e^{-K_1 y}\,dy \tag{39}$$

where

$$K_1 = \frac{b\bar{H}_{o,w}\bar{l}_o}{wc_p(1+\lambda)} \tag{62}$$

$$K_2 = -\frac{b\omega^2 w(1+\bar{\lambda})}{Jg\bar{H}_{o,w}\bar{l}_o} \tag{63}$$

$$K_3 = \frac{\omega^2 w(1+\bar{\lambda})}{Jg\bar{H}_{o,w}\bar{l}_o}(b+r_h) - \frac{\bar{c}_p\omega^2 w^2(1+\bar{\lambda})^2}{Jg\bar{H}_{o,w}{}^2\bar{l}_o{}^2} \tag{64}$$

The integration constant K may be determined from the boundary conditions at either end of the blade coolant passage. At the blade tip,

$$y = 0$$

and

$$T'' = T_T''$$

The substitution of these values into equation (39) yields the following value for the integration constant:

$$K = T''_T - K_3 \tag{65}$$

At the blade root

$$y = 1$$

and

$$T'' = T''_h$$

so that the substitution of these values into equation (39) results in the following value of the integration constant:

$$K = e^{-K_1}\left(T''_h - K_2 - K_3\right) + K_1\int_0^1 T_{g,e}\, e^{-K_1 y}\, dy \tag{65a}$$

<u>Closed-form solution of energy equation when $T_{g,e}$ is assumed constant.</u> - In addition to the assumption that the parameters are

constant and equal to a mean value in equation (60), an unweighted integrated mean value of $T_{g,e}$ (fig. 3(g)) may be employed. Equation (39) then becomes

$$T'' = Ke^{K_1 y} + K_2 y + K_3 + \overline{T}_{g,e} \tag{40}$$

As in the previous section, the value of the constant K may be evaluated at either end of the blade coolant passage. When the value of T''_T is known $(y = 0)$

$$K = T''_T - K_3 - \overline{T}_{g,e} \tag{41}$$

and when the value of T''_h is known $(y = 1)$

$$K = e^{-K_1}\left(T''_h - K_2 - K_3 - \overline{T}_{g,e}\right) \tag{41a}$$

APPENDIX C

DERIVATION OF MOMENTUM EQUATION

The general form of the momentum equation is (reference 11, p. 117)

$$\frac{dp}{\rho g} - F_b\,dr + \frac{W\,dW}{g} + \frac{dp_{fr}}{\rho g} = 0 \tag{4}$$

The infinitesimal pressure drop due to friction is

$$dp_{fr} = \frac{4f\rho W^2}{2D_h}\,dr \tag{66}$$

In a rotating passage, F_b is the centrifugal force as before so that

$$F_b\,dr = \frac{\omega^2 r\,dr}{g} \tag{44}$$

The substitution of equations (66) and (44) into equation (4) results in the following form of the momentum equation:

$$\frac{dp}{\rho} - \omega^2 r\,dr + W\,dW + \frac{2fW^2 dr}{D_h} = 0 \tag{67}$$

From the definition of Mach number,

$$W^2 = M^2 a^2$$

$$= M^2 \gamma gRT \tag{68}$$

When equations (1) and (68) are combined

$$\rho W^2 = \gamma p M^2 \tag{69}$$

and

$$\rho = \frac{\gamma p M^2}{W^2} \tag{70}$$

The substitution of the expression for ρ (equation (70)) into equation (67) yields

$$\frac{dp}{p} - \frac{\gamma M^2}{W^2}\omega^2 r\ dr + \frac{\gamma M^2}{2W^2} 2W\ dW + \frac{\gamma\ M^2}{2}\frac{4f\ dr}{D_h} = 0$$

or

$$\frac{dp}{p} + \frac{\gamma\ M^2}{2}\frac{d(W^2)}{W^2} + \frac{\gamma\ M^2}{2}\left(\frac{4f\ dr}{D_h} - \frac{\omega^2 r\ dr}{W^2/2}\right) = 0. \tag{71}$$

This result agrees with equation (9) of reference 1. For a numerical solution, equation (71) must be transformed into an equation similar to equation (19) of reference 1, in which the only unknown quantity is M^2. This equation can then be solved for M^2 or M; pressures, velocities, densities, and so forth at any point along the passage can then be found by means of equations (21) to (26) of reference 1.

In order to make the transformation of equation (71), equation (1) is written in the form

$$\log_e p = \log_e \rho + \log_e g + \log_e R + \log_e T \tag{72}$$

and differentiated so as to result in

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} \tag{73}$$

A similar operation on equation (2) yields

$$0 = \frac{d\rho}{\rho} + \frac{dW}{W} + \frac{dA}{A} \tag{74}$$

The substitution of equation (74) into equation (73) results in

$$\frac{dp}{p} = \frac{dT}{T} - \frac{dW}{W} - \frac{dA}{A} \tag{75}$$

From the logarithmic differentiation of equation (68),

$$\frac{d(W^2)}{W^2} = \frac{dM^2}{M^2} + \frac{dT}{T} \tag{76}$$

or

$$\frac{2W\ dW}{W^2} = \frac{2M\ dM}{M^2} + \frac{dT}{T}$$

Finally,

$$\frac{dW}{W} = \frac{dM}{M} + \frac{1}{2}\frac{dT}{T} \tag{77}$$

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The substitution of the equations (75) to (77) into equation (72) yields

$$\frac{dT}{T} - \frac{dM}{M} - \frac{1}{2}\frac{dT}{T} - \frac{dA}{A} + \frac{\gamma M^2}{2}\left(\frac{dM^2}{M^2} + \frac{dT}{T}\right) + \frac{\gamma M^2}{2}\left(\frac{4f\ dr}{D_h} - \frac{\omega^2 r\ dr}{W^2/2}\right) = 0 \tag{78}$$

or

$$\frac{dT}{T}\left(\frac{1}{2} + \frac{\gamma M^2}{2}\right) + \frac{\gamma M^2}{2}\left(\frac{4f\ dr}{D_h} - \frac{\omega^2 r\ dr}{W^2/2}\right) = \frac{dM}{M} + \frac{dA}{A} - \frac{\gamma M^2}{2}\frac{dM^2}{M^2} \tag{79}$$

The relation between the total and static temperature is

$$T'' = T\left(1 + \frac{\gamma-1}{2}M^2\right) \tag{80}$$

which when logarithmically differentiated results in

$$\frac{dT}{T} = \frac{dT''}{T''} - \frac{\frac{\gamma-1}{2}M^2}{1 + \frac{\gamma-1}{2}M^2}\frac{dM^2}{M^2} \tag{81}$$

The substitution of the value of dT/T from equation (81) into equation (79) yields

$$\frac{dM^2}{M^2} = \frac{(1 + \gamma M^2)\left(1 + \frac{\gamma-1}{2}M^2\right)}{(1 - M^2)}\frac{dT''}{T''} + \frac{\gamma M^2\left(1 + \frac{\gamma-1}{2}M^2\right)}{(1 - M^2)}\left(\frac{4f\ dr}{D_h} - \frac{\omega^2 r\ dr}{W^2/2}\right) - \frac{2\left(1 + \frac{\gamma-1}{2}M^2\right)}{(1 - M^2)}\frac{dA}{A} \tag{82}$$

If in equation (69) both sides are multiplied by 1/2 and the value of T is replaced by its equivalent from equation (80), the following relation results:

$$\frac{W^2}{2} = \frac{\frac{\gamma g R T'' M^2}{2}}{1 + \frac{\gamma-1}{2} M^2} \tag{83}$$

The substitution of the value of $W^2/2$ from equation (83) into equation (82) yields the following results when both sides are multiplied by M^2, as in reference 7.

$$dM^2 = \frac{M^2 (1 + \gamma M^2)\left(1 + \frac{\gamma-1}{2} M^2\right)}{(1 - M^2)} \frac{dT''}{T''} +$$

$$\frac{\gamma M^4 \left(1 + \frac{\gamma-1}{2} M^2\right)}{(1 - M^2)} \left[\frac{4f\ dr}{D_h} - \frac{2\omega^2 r\ dr \left(1 + \frac{\gamma-1}{2} M^2\right)}{\gamma g R T'' M^2}\right] -$$

$$\frac{2M^2 \left(1 + \frac{\gamma-1}{2} M^2\right)}{(1 - M^2)} \frac{dA}{A} \tag{84}$$

In order to convert r to a dimensionless quantity, it is convenient to substitute

$$r = r_h + (1-y)b \tag{58}$$

into equation (84). The incorporation of this substitution into equation (84) results in the final form of the working equation

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$$\frac{dM^2}{dy} = \frac{I_T}{T''}\frac{dT''}{dy} - I_f\left[\frac{4fb}{D_h} - \frac{I_R}{T''}\frac{2\omega^2 r_h b}{gR}\left(1 + \frac{b}{r_h} - y\,\frac{b}{r_h}\right)\right] + \frac{dA}{dy}\frac{I_A}{A} \tag{9}$$

where

$$I_R = \frac{1 + \frac{\gamma-1}{2}M^2}{\gamma M^2} \tag{11}$$

$$I_A = -\frac{2M^2\left(1 + \frac{\gamma-1}{2}M^2\right)}{1 - M^2} \tag{12}$$

$$I_f = \frac{\gamma M^4\left(1 + \frac{\gamma-1}{2}M^2\right)}{1 - M^2} \tag{13}$$

$$I_T = \frac{M^2(1 + \gamma M^2)\left(1 + \frac{\gamma-1}{2}M^2\right)}{1 - M^2} \tag{14}$$

as in reference 7. The values of I_A, I_f, I_R, and I_T may be determined as functions of M and γ from table I.

APPENDIX D

DERIVATION OF FORMULA FOR TIP MACH NUMBER

The value of the Mach number at the coolant-passage exit is 1199
determined from the continuity equation (2) written in the form:

$$w^2 = A^2 W^2 \rho^2 g^2 \tag{85}$$

and the definition of the Mach number

$$M^2 = \frac{W^2}{\gamma g R T} \tag{86}$$

The elimination of W^2 from equations (85) and (86) results in

$$M^2 = \frac{w^2}{A^2 \rho^2 g^3 \gamma R T} \tag{87}$$

The value of ρ from equation (1) is then introduced into equation (87) so that

$$M^2 = \frac{w^2 R T}{A^2 g \gamma p^2} \tag{88}$$

Finally, the value of T from equation (80) is introduced into equation (88) with the result that

$$I_c = \gamma M^2 \left(1 + \frac{\gamma - 1}{2} M^2\right) = \frac{w^2 R T''}{A^2 g p^2} \tag{18}$$

The value of the tip Mach number corresponding to the right side of equation (18) can be determined from table I.

APPENDIX E

OUTSIDE HEAT-TRANSFER COEFFICIENT

A limited amount of data exists on the outside heat-transfer coefficients for impulse turbine blades. A series of investigations was therefore conducted with hot air (300° F) flowing across cooled blades. The blades were of the symmetrical impulse type arranged in a straight static cascade.

Theory indicates (reference 18) that in order to correlate heat-transfer data the temperature ratio T_B/T must be included as one of the dimensionless parameters with Nu, Re, and Pr. The previously mentioned experimental data were found to correlate on the basis of the wall temperature; equation (3) is the relation recommended from the data. The data, which included the range $2 \times 10^4 < Re < 10^5$, may not apply under conditions differing widely from those investigated.

$$\frac{H_{o,w}\, D_{h,o}}{k_{g,w}} = 0.75 \left(\frac{\rho_{g,w}\, W_g D_{h,o}}{\mu_{g,w}}\right)^{0.53} \left(Pr_{g,w}\right)^{1/3} \tag{30}$$

In equation (30), the fluid properties are based on T_B, the value of which must be assumed in order to initiate the computations. The velocity W_g and the density $\rho_{g,w}$ are determined at the rotor entrance. The value of W_g is obtained from equation (102) and the value of $\rho_{g,w}$ is obtained from the relation

$$\bar{\rho}_{g,w} = \frac{\bar{p}_{ex}}{gRT_B} \tag{31}$$

The value of $D_{h,o}$ utilized in equation (30) is the outside perimeter of the blade divided by π. The variation of p_{ex} along the blade is neglected because it is small compared with the variation of T_B.

APPENDIX F

INSIDE HEAT-TRANSFER COEFFICIENT

No data exist for the correlation of the heat-transfer coefficient on the inside of rotating turbine blades. The prediction of the inside heat-transfer coefficients must therefore be accomplished by the modification of correlations for heat transfer in pipes.

Like the friction coefficient, the inside heat-transfer coefficient is a function of the distance from the entrance to the passage in the region of nonstabilized flow. Variations of the coolant-passage inside heat-transfer coefficient, however, affect the Mach number distribution as calculated from equation (9) much less than f; the variations of the inside heat-transfer coefficient due to entrance effects are therefore neglected herein.

The dimensionless analysis of the inside heat-transfer coefficient for forced convection indicates that the Nusselt and Grashof numbers, among other dimensionless numbers, are related. The Grashof number is defined as

$$Gr = \frac{b^3 \rho^2 \beta (\omega^2 r)(T_B - T_e)}{\mu^2}$$

Because the acceleration $\omega^2 r$ appears in Gr, this dimensionless parameter may prove to have a significant effect on the inside heat-transfer coefficient for a rotating passage. In stationary passages, the effect of Gr on Nu is neglected where the acceleration is g. In the rotating passage, however, the acceleration $\omega^2 r$ is from 10,000 to 50,000 g. No reliable correlations of the inside heat-transfer coefficient for forced convection that include Gr exist. The effect of Gr on the inside heat-transfer coefficient is therefore neglected herein. In the numerical example, a numerical estimate indicated that the free convection airspeeds associated with the Grashof number effects are less than 10 percent of those associated with the Reynolds number.

In reference 11 (p. 168) the following relation is presented for the flow of fluids in pipes:

$$Nu = 0.023 \, (Re)^{0.8} \, (Pr)^{0.4} \tag{89}$$

Equation (89) results in reasonably accurate values of the inside heat-transfer coefficients for heat-exchange equipment when the fluid properties in the dimensionless numbers are evaluated at the fluid static temperature. In such equipment, the fluid velocity and the difference between the fluid static temperature and the wall temperature are usually small. If this temperature difference and the fluid velocity are large, as in aircraft heat exchangers or blade coolant passages, the constants 0.8 and 0.023 in equation (89) have new values and a different value for each wall temperature or each temperature difference. In reference 13, a satisfactory correlation for a range of wall temperatures and a corresponding range of temperature differences is found to exist when equation (89) is employed if all the fluid properties except W are based on the wall temperature. In addition, it is pointed out in reference 13 that the Prandtl number affects the correlation to such a small extent that Pr may be neglected in the correlation. The following relation is therefore recommended for a gas flowing inside a passage at a high velocity with a high temperature difference between the effective gas temperature and the wall temperature (reference 13):

$$\frac{H_{i,w}D_h}{k_w} = 0.019\left(\frac{\rho_w\ WD_h}{\mu_w}\right)^{0.8} \qquad (35)$$

For the present purpose, an equivalent form of equation (35) suggested by an author of reference 13 will be used.

$$\frac{H_{i,w}D_h}{k} = 0.019\left(\frac{W}{A}\frac{D_h}{\mu_w}\right)^{0.8} \qquad (90)$$

If $k \propto T^{0.8}$, equations (35) and (90) will be the same. They will be approximately the same over a moderate temperature range if the exponent of T is between 0.7 and 0.9.

For a finned blade (fig. 1(d)), $H_{i,w}$ determined from equation (35) is based on the total heat-transfer surface (including the fins) and the temperature difference between T_e and the average of the fin temperature and the blade temperature T_B. For application in the energy equation (8), it is convenient to define for a finned blade an inside heat-transfer coefficient H_f, which is based on the heat-transfer area of the equivalent hollow blade (fig. 1(b)) and the temperature difference $(T_B - T_e)$. In reference 14, a relation is developed between $H_{1,w}$ and H_f for fins on the outside surface of a circular cylinder. If in this relation the radius of the cylinder is taken equal to infinity

$$H_f = \frac{H_{1,w}}{m + \tau}\left(\frac{2 \tanh \varphi L_f}{\varphi} + m\right) \tag{36}$$

When finned blades are being analyzed, H_i is replaced by H_f in the energy equation (8).

The division of equation (90) by equation (90) written in terms of reference values yields

$$\frac{H_{1,w}}{\overline{H}_{1,w}} = \left(\frac{\overline{D}_h}{D_h}\right)^{0.2}\left(\frac{\overline{\mu}_w}{\mu_w}\right)^{0.8}\frac{k}{\overline{k}}\left(\frac{\overline{A}}{A}\right)^{0.8} \tag{91}$$

In order to reduce this equation to geometry and temperature factors, the following relations are substituted into equation (91):

$$\frac{\overline{\mu}_w}{\mu_w} = \left(\frac{\overline{T}_B}{T_B}\right)^{0.7} \tag{92}$$

$$\frac{k}{\overline{k}} = \left(\frac{T''}{\overline{T}''}\right)^{0.7} \tag{93}$$

$$\frac{\overline{D}_h}{D_h} = \frac{\overline{A}}{A}\,\frac{l_f}{\overline{l}_f} \tag{94}$$

Equation (94) is obtained from the definition of $D_h = \left(\frac{4 \times \text{area}}{\text{perimeter}}\right)$; l_f is the total wetted perimeter of walls and fins.

The substitution yields

$$\frac{H_{i,w}}{\bar{H}_{i,w}} = \frac{\bar{A}}{A}\left(\frac{l_f}{\bar{l}_f}\right)^{0.2}\left(\frac{\bar{T}_B}{T_B}\right)^{0.56}\left(\frac{T''}{\bar{T}''}\right)^{0.7} \tag{95}$$

Substitution of equation (95) into equations (36) and (36) written for reference values yields

$$\frac{H_f l_i}{\bar{H}_f \bar{l}_i} = \frac{l_i}{\bar{l}_i}\,\frac{\bar{A}}{A}\left(\frac{l_f}{\bar{l}_f}\right)^{0.2}\left(\frac{T''}{\bar{T}''}\right)^{0.7}\left(\frac{\bar{T}_B}{T_B}\right)^{0.56}\frac{m + \dfrac{2\tanh \varphi L_f}{\varphi}}{m + \dfrac{2\tanh \bar{\varphi}\bar{L}_f}{\bar{\varphi}}} \tag{37}$$

The relation for φ in terms of reference values is

$$\frac{\varphi}{\bar{\varphi}} = \sqrt{\frac{H_{i,w}}{\bar{H}_{i,w}}} \tag{96}$$

and from equations (95) and (96)

$$\frac{\varphi L_f}{\bar{\varphi}\bar{L}_f} = \frac{L_f}{\bar{L}_f}\left(\frac{l_f}{\bar{l}_f}\right)^{0.1}\left(\frac{\bar{A}}{A}\right)^{0.5}\left(\frac{T''}{\bar{T}''}\right)^{0.35}\left(\frac{\bar{T}_B}{T_B}\right)^{0.28} \tag{38}$$

APPENDIX G

NUMERICAL EXAMPLE USING MEAN VALUES FOR PARAMETERS

For many purposes a closed-form solution of the energy equation utilizing mean values for all parameters may be employed in the determination of the flow characteristics of the cooling air in blade coolant passages. In this appendix a numerical example is presented in which all the blade dimensions required in the computations are those at the midpoint of the blade $(y = 0.5)$ and an integrated mean value of $T_{g,e}$ is employed. The data required are the same as presented in the section Assumed Conditions for Numerical Example. In the preceding numerical example, reference values for $H_{o,w}l_o$ and λ were computed based on 1199

$$\bar{T}'' = 900^\circ \text{ R}$$

$$\bar{T}_B = 1300^\circ \text{ R}$$

and the blade dimensions at midspan, which are

$$\bar{H}_{o,w}\bar{l}_o = 0.01162$$

$$\bar{\lambda} = 1.296$$

These reference values may be used as a first approximation for the solution of the energy equation written in the form of equation (40). The constants for equation (40) are

$$K_1 = \frac{b\,\bar{H}_{o,w}\bar{l}_o}{w\,\bar{c}_p(1+\bar{\lambda})} = \frac{(0.3)(0.01162)}{(0.01689)(0.246)(2.296)} = 0.365$$

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$$K_2 = -\frac{b\omega^2 w(1+\overline{\lambda})}{gJ\overline{H}_{o,w}\overline{l}_o} = -\frac{(0.3)(796)^2(0.01689)(2.296)}{(32.17)(778)(0.01162)}$$

$$= -25.3$$

$$K_3 = \frac{\omega^2 w(1+\overline{\lambda})(b+r_h)}{gJ\overline{H}_{o,w}\overline{l}_o} - \frac{\overline{c}_p\omega^2 w^2(1+\overline{\lambda})^2}{gJ\overline{H}_{o,w}{}^2\overline{l}_o{}^2}$$

$$= \frac{(796)^2(0.01689)(2.296)(0.3 + 1.117)}{(32.17)(778)(0.01162)} - \frac{0.246(796)^2(0.01689)^2(2.296)^2}{(32.17)(778)(0.01162)^2}$$

$$= 50.4$$

$$K = T''_T - K_3 - \overline{T}_{g,e} = 1160 - 50.4 - 1967 = -857.4$$

The trial form of equation (40) employing the values of the constants just computed is

$$T'' = -857.4\ e^{0.365} - 25.3\ y + 50.4 + 1967 \tag{97}$$

The value

$$\overline{T}'' = 976^\circ\ R$$

results when $y = 0.5$ is substituted into equation (97) and the corresponding value of $\overline{T}_B$ is

$$\overline{T}_B = \frac{\overline{\lambda}\,\overline{T}_{g,e} + \overline{T}''}{1 + \overline{\lambda}} = \frac{(1.296)(1967) + 976}{2.296} = 1535^\circ\ R$$

Inasmuch as the assumed temperatures $\bar{T}''$ and $\bar{T}_B$ do not agree with the computed temperatures, the values of $\bar{H}_{o,w}\bar{l}_o$ and $(1 + \bar{\lambda})$ may be recomputed based on the values obtained for $\bar{T}''$ and $\bar{T}_B$. When another trial had been made it was found that if the values 1199

$$\bar{T}'' = 980^\circ \text{ R}$$

$$\bar{T}_B = 1565^\circ \text{ R}$$

were assumed for the computation of the heat-transfer coefficients, the following results were obtained:

$$\bar{H}_{o,w}\bar{l}_o = 0.01260 \qquad K_3 = 50.7$$

$$1 + \bar{\lambda} = 2.441 \qquad K = -857.7$$

$$K_1 = 0.373 \qquad \bar{T}'' = 972^\circ \text{ R}$$

$$K_2 = -24.9 \qquad \bar{T}_B = 1559^\circ \text{ R}$$

Because the computed values of $\bar{T}''$ and $\bar{T}_B$ agreed closely with the assumed values, the constants just presented were employed in equation (40) in order to determine the errors in $\bar{T}''$ presented in figure 8.

The distributions of M, W, and p along the blade coolant passage were computed in the same manner as in the foregoing numerical example.

APPENDIX H

EFFECTIVE TEMPERATURE

The effective temperature T_e is defined as the temperature a body would assume with no heat transfer to or from it if the body were placed in a high-velocity stream with the static temperature T. The value of T_e is approximately $T + 0.9(T'' - T)$.

No relation has heretofore been presented for obtaining $\overline{T}_{g,e}$ when only the mean total temperature of the gas at the combustion-chamber exit is specified; a relation between $\overline{T}_{g,e}$ and Λ_o will therefore be derived.

If it is assumed that the nozzles upstream of the turbine are uncooled, the ideal temperature at the nozzle exit may be determined from the adiabatic law:

$$\overline{T}_{g,ex,id} = \overline{T}''_{g,in}\left(\frac{\overline{p}_{g,ex}}{\overline{p}''_{g,in}}\right)^{\frac{\gamma_g-1}{\gamma_g}} \tag{98}$$

The average temperature of the combustion gas at the nozzle exit is

$$\overline{T}_{g,ex,av} = \overline{T}''_{g,in} - \eta\left(\overline{T}''_{g,in} - \overline{T}_{g,ex,id}\right) \tag{99}$$

where η is the nozzle (stator-ring) efficiency. When the velocity of the fluid entering the nozzles is neglected, the nozzle exit velocity may be determined from the relation

$$\left(\overline{V}_{g,ex}\right)^2 = 2Jgc_{p,g}\left(\overline{T}''_{g,in} - \overline{T}_{g,ex,av}\right) \tag{100}$$

Finally, the combination of equations (98) to (100) results in the expression for the square of the combustion-gas velocity at the nozzle exit.

$$\left(\overline{V}_{g,ex}\right)^2 = 2Jgc_{p,g}\,\eta\,\overline{T}_{g,in}\left[1-\left(\frac{\overline{p}_{g,ex}}{\overline{p}''_{g,in}}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right] \tag{101}$$

For a nozzle, η is usually about 0.9.

The velocity of the combustion gas relative to the rotating blades at the rotor entrance is obtained from the solution of the equation

$$\overline{W}_g = \sqrt{\left(\overline{V}_{g,ex}\right)^2 + \overline{u}^2 - 2\overline{V}_{g,ex}\,\overline{u}\cos\alpha} \tag{102}$$

which is obtained from the velocity diagram of figure 2(b). The value of $\overline{u}$ in equation (102) is given by

$$\overline{u} = 2\pi N\overline{r}$$

where

$$\overline{r} = \frac{(r_T + r_h)}{2}$$

The effective combustion-gas temperature at the rotor entrance is obtained from the definition of Λ_o

$$\Lambda_o = \frac{T_{g,e} - T_g}{T''_g - T_g} = \frac{2Jgc_{p,g}(T_{g,e} - T_g)}{W_g^2} \tag{103}$$

$$\overline{T}_{g,e} = \overline{T}_{g,ex} + \Lambda_o\frac{W_g^2}{2Jgc_{p,g}} \tag{104}$$

where the value of $\overline{T}_{g,ex}$ is equivalent to T_g in equation (103).

APPENDIX I

RECOVERY COEFFICIENT

The recovery coefficient enters the analysis of heat transfer to high-velocity streams because the temperature difference causing the heat transfer to such streams is $(T_B - T_e)$ rather than $(T_B - T)$, as for low-velocity streams.

According to reference 19, the recovery coefficient Λ is a function of Re, Pr, and M and is defined for the outside of a blade as

$$\Lambda_o = \frac{T_{g,e} - T_g}{T''_g - T_g} = \frac{2Jgc_{p,g}(T_{g,e} - T_g)}{W_g^{\,2}} \tag{103}$$

The corresponding recovery coefficient for the inside of the blade is defined as

$$\Lambda_i = \frac{T_e - T}{T'' - T} \tag{15}$$

It has been established both by theory and by experiment (reference 20) that for a laminar layer of air flowing along a flat plate with no pressure gradient the recovery coefficient is substantially independent of Re and M and may be represented by the equation

$$\Lambda = (Pr)^{1/2} \tag{105}$$

Data from the same reference indicate that, for a turbulent boundary layer of air in subsonic flow over a flat plate, values of Λ increase from the value of 0.84 given by equation (105) to 0.89 as Re increases.

The values of Λ obtained for turbine blades confirm the relation

$$\Lambda = (Pr)^{1/3} \tag{106}$$

presented in reference 21 for a turbulent boundary layer along a flat plate.

A possible mean value for the recovery coefficient on the outside of a turbine blade Λ_o based on the relative portions of the perimeter of the blade having laminar and turbulent boundary layers is

$$\Lambda_o = (x)(Pr)^{1/2} + (1-x)(Pr)^{1/3} \tag{107}$$

where

x portion of blade perimeter having laminar boundary layer.

1-x portion of blade perimeter having turbulent boundary layer.

In reference 22, experimental data are reported on the recovery coefficient for the inside of a pipe having a turbulent boundary layer. The average value reported is $\Lambda_i = 0.88$. For the present report where only subsonic air flows are considered, it has already been mentioned that the value of Λ has but a slight influence on the result. It will therefore be sufficient to assume that $\Lambda_o = \Lambda_i = 0.9$.

APPENDIX J

FRICTION COEFFICIENT

Considerable experimental data exist for the determination of the friction coefficient of fully developed flow in a passage without heat transfer. Few data are currently available, however, for the evaluation of the friction coefficient in the presence of heat transfer. In addition, entrance effects result in higher apparent friction coefficients than are experienced in fully developed turbulent flow regardless of the presence of heat transfer. The apparent coefficient also includes profile losses.

The friction coefficient in the absence of heat transfer is defined as

$$f = \frac{D_h}{2\rho W^2} \frac{dp_{fr}}{dr} \tag{108}$$

(This equation gives only the local value. In a passage of finite length, the average value of f is defined by equation (9) and is so chosen in this case that the over-all pressure drop through the passage is correct, although intermediate values may not be.) In equation (108), dp_{fr} is the differential pressure drop due to friction in the length of passage dr. Coolant passages in air-cooled turbine blades are usually noncircular so that the hydraulic diameter instead of the tube diameter occurs in equation (108). The hydraulic diameter of a passage (fig. 1) is equal to four times the flow area divided by the wetted perimeter.

Throughout most of the passage, the flow is turbulent. According to von Kármán's formula, reference 11 (p. 119), the correlation between f and Re for fully developed turbulent flow in smooth pipes may be expressed as

$$\frac{1}{\sqrt{f_{vK}}} = 4.0 \log \left(Re \sqrt{f_{vK}}\right) - 0.40 \tag{109}$$

A plot of equation (109), which facilitates the determination of f from Re, is presented in figure 6. In reference 11 (p. 119) another correlation between f and Re is presented for a limited range of Re $(5000 \leqq Re \leqq 200{,}000)$ and is

$$f = \frac{0.046}{(Re)^{0.2}} \tag{110}$$

The value of Re required in equations (109) and (110) is determined from the equation

$$Re = \frac{\rho W D_h}{\mu} = \frac{w D_h}{A \mu g} \tag{111}$$

The friction coefficient is determined less accurately from equations (109) and (110) as the rate of heat transfer increases. In reference 11 (p. 121) it is indicated that, for small temperature differences between the fluid and the passage wall, the friction coefficient in the presence of heat transfer may be satisfactorily determined from equation (109) if the mean film temperature is employed in the determination of μ in equation (111). The mean film temperature denoted here is the average of the passage-wall temperature and the fluid static temperature. It is suggested in reference 13 that for large temperature differentials between the wall temperature and the fluid static temperature (where the ratio of the wall temperature to the fluid bulk temperature is about 2) the effect of heat transfer on the friction coefficient may be accounted for through the employment of values of ρ and μ in equation (111) and ρ in equation (108) based on the wall temperature. Equations (108) and (111), respectively, then take the forms

$$f = \frac{D_h}{2\rho W^2} \frac{\rho}{\rho_w} \frac{dp_{fr}}{dr} \tag{112}$$

$$Re_w = \frac{\rho_w W D_h}{\mu_w} \tag{113}$$

An attempt was made to correlate the data of reference 23 on the foregoing basis. The values of f in the presence of heat transfer thus determined were found to be considerably lower than the actual values of f. Currently, it would therefore seem that the data are insufficient to establish the theory that the value of f in the presence of heat transfer may be correlated on a basis of the wall temperature. Examination of the data of reference 23 indicates, however, that the value of f required to satisfy equation (68) with heat transfer is not greater than that (except for exit Mach numbers less than 0.2) given by equation (109) when μ and ρ are determined at the stream temperature of the passage exit.

The results of reference 24 indicate that values of f larger and smaller than predicted by equation (109) occur in the entrance section of a passage. In order to avoid underestimating the pressure-drop values of f obtained from the von Kármán relation, equation (109) may be corrected for entrance conditions by employing the correction factors (reference 24) presented in the following table:

$\frac{b}{D_h}$	$\frac{f}{f_{vK}}$
5	1.43
10	1.35
20	1.20
30	1.10
40	1.05

In this table b refers to the total distance measured from the entrance of the coolant passage and not merely from the root of the blade where heat transfer begins.

The recommended procedure for computing f is to calculate Re corresponding to the exit conditions, determine f_{vK} from equation (109) or figure 6, and then apply the correction factors of the preceding table where required.

When this procedure is employed for the determination of f, the fluid properties may be evaluated at T''. The small error that will result from this approximation can be seen from the combination of equations (110) and (111).

$$f = \frac{0.046}{\left(\frac{w}{A}\frac{D_h}{g\mu}\right)^{0.2}} \tag{114}$$

From equation (114), f is seen to be rather insensitive to temperature because μ, which varies approximately with $T^{0.7}$, appears to the 0.2 power.

As an example of this procedure and in connection with the numerical examples of this report, the friction coefficient will be computed. At the tip of the blade coolant passage, from figures 3(a) and 3(b)

$$A = 0.000194$$

$$D_h = 0.0044$$

and from reference 14 at a value of $T'' = 1160^\circ$ R

$$\mu = 0.66 \times 10^{-6}$$

so that

$$Re = \frac{w}{gA}\frac{D_h}{\mu} = \frac{(0.01689)(0.0044)}{(32.17)(0.000194)(0.66 \times 10^{-6})} = 17,800$$

$$f_{vK} = \frac{0.046}{(Re)^{0.2}} = \frac{0.046}{(17,800)^{0.2}} = 0.0065$$

The length-diameter ratio of the passage is

$$\frac{b}{D_h} = \frac{0.3}{0.0044} = 68$$

No correction for entrance effects is required.

REFERENCES

1. Shapiro, Ascher H., and Hawthorne, W. R.: The Mechanics and Thermodynamics of Steady One-Dimensional Gas Flow. Jour. Appl. Mech., vol. 14, no. 4, Dec. 1947, pp. A317-A336.

2. Nielsen, Jack N.: High-Altitude Cooling. III - Radiators. NACA ARR L4I11b, 1944.

3. Becker, John V., and Baals, Donald D.: Simple Curves for Determining the Effects of Compressibility on Pressure Drop through Radiators. NACA ACR L4I23, 1944.

4. Habel, Louis W., and Gallagher, James J.: Tests to Determine the Effect of Heat on the Pressure Drop through Radiator Tubes. NACA TN 1362, 1947.

5. Turner, L. Richard, Addie, Albert N., and Zimmerman, Richard H.: Charts for the Analysis of One-Dimensional Steady Compressible Flow. NACA TN 1419, 1948.

6. Valerino, Michael F.: Generalized Charts for Determination of Pressure Drop of a High-Speed Compressible Fluid in Heat-Exchanger Passages. I - Air Heated in Smooth Passages of Constant Area with Constant Wall Temperature. NACA RM E8G23, 1948.

7. Edelman, G. M., and Shapiro, Ascher H.: Tables for Numerical Solution of Problems in the Mechanics and Thermodynamics of Steady One-Dimensional Gas Flow without Discontinuities. Jour. Appl. Mech., vol. 14, no. 4, Dec. 1947, pp. A344-A351.

8. Stodola, A.: Steam and Gas Turbines. Vol. II. McGraw-Hill Book Co., Inc., 1927, p. 1259. (Reprinted, Peter Smith (New York), 1945.)

9. Brown, W. Byron, and Livingood, John N. B.: Cooling of Gas Turbines. III - Analysis of Rotor and Blade Temperatures in Liquid-Cooled Gas Turbines. NACA RM E7B11c, 1947.

10. Wolfenstein, Lincoln, Maxwell, Robert L., and McCarthy, John S.: Cooling of Gas Turbines. V - Effectiveness of Air Cooling of Hollow Blades. NACA RM E7B11e, 1947.

11. McAdams, William H.: Heat Transmission. McGraw-Hill Book Co., Inc., 2d ed., 1942, pp. 117, 119, 121, 168, 232, 242.

12. Moulton, Forest Ray: Differential Equations. The Macmillan Co. (New York), 1930, pp. 213-222.

13. Lowdermilk, Warren H., and Grele, Milton D.: Heat Transfer from High-Temperature Surfaces to Fluids. II - Correlation of Heat-Transfer and Friction Data for Air Flowing in Inconel Tube with Rounded Entrance. NACA RM E8L03, 1949.

14. Ellerbrock, Herman H., Jr., and Biermann, Arnold E.: Surface Heat-Transfer Coefficients of Finned Cylinders. NACA Rep. 676, 1939.

15. Keenan, Joseph H., and Kaye, Joseph: Thermodynamic Properties of Air. John Wiley & Sons, Inc., 1945, p. 36.

16. Ellerbrock, Herman H., Jr., Woislo, Chester R., and Dexter, Howard E.: Analysis, Verification, and Application of Equations and Procedures for Design of Exhaust-Pipe Shrouds. NACA TN 1495, 1947.

17. Kells, Lyman M.: Elementary Differential Equations. McGraw-Hill Book Co., Inc., 2d ed., 1935, p. 50.

18. Jakob, Max: Heat Transfer. Vol. I. John Wiley & Sons, Inc., 1949, p. 497.

19. Johnson, H. A., and Rubesin, M. W.: Aerodynamic Heating and Convective Heat Transfer - Summary of Literature Survey. Trans. ASME, vol. 71, no. 5, July 1949, pp. 447-456.

20. Eckert, and Weise: The Temperature of Uncooled Turbine Blades in a Fast Stream of Gas. Reps. & Trans. No. 39, British M.A.P., March 1946. (Distributed in U.S. by British J.I.O.A. (Washington, D.C.), June 26, 1946.)

21. Ackermann, G.: Plattenthermometer in Strömung mit grosser Geschwindigkeit und turbulenter Grenzschicht. Forschung auf dem Gebiete des Ingenieurwesens, Bd. 13, Heft 6, Nov./Dez. 1942, S. 226-234.

22. McAdams, William H., Nicolai, Lloyd A., and Keenan, Joseph H.: Measurements of Recovery Factors and Coefficients of Heat Transfer in a Tube for Subsonic Flow of Air. NACA TN 985, 1945.

23. Gallagher, James J., and Habel, Louis W.: Experimental Data Concerning the Effect of High Heat-Input Rates on the Pressure Drop through Radiator Tubes. NACA RM L8F18, 1948.

24. Shapiro, Ascher H., and Smith, R. Douglas: Friction Coefficients in the Inlet Length of Smooth Round Tubes. NACA TN 1785, 1948.

TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	^1I_A		^2I_f		^3I_T	
		γ		γ		γ	
		1.37	1.40	1.37	1.40	1.37	1.40
0.002	0.000004	-0.0000080	00080	0.00319×10^{-8}	00224	0.00400×10^{-3}	00400
.004	00016	- 00320	00320	00351×10^{-7}	00358	00160×10^{-2}	00160
.006	00036	- 00720	00720	00178×10^{-6}	00181	00360	00360
.008	00064	- 01280	01280	00561	00573	00640	00640
0.010	0.000100	-0.000200	00200	0.00137×10^{-5}	00140	0.00100×10^{-2}	00100
.012	00144	- 00288	00288	00284	00290	00144	00144
.014	00196	- 00392	00392	00526	00538	00196	00196
.016	00256	- 00512	00512	00898	00918	00256	00256
.018	00324	- 00648	00648	01439	01470	00324	00324
0.020	0.000400	-0.000800	00800	0.02193×10^{-5}	02241	0.00400×10^{-2}	00400
.022	00484	- 00969	00969	03211	03281	00484	00485
.024	00576	- 01153	01153	04548	04648	00577	00577
.026	00676	- 01353	01353	06266	06403	00677	00677
.028	00784	- 01569	01569	08429	08613	00786	00786
0.030	0.000900	-0.001802	01802	0.01111×10^{-4}	01135	0.00902×10^{-2}	00902
.032	01024	- 02050	02051	01438	01470	01027	01027
.034	01156	- 02315	02315	01834	01873	01159	01159
.036	01296	- 02596	02596	02305	02355	01300	01300
.038	01444	- 02893	02893	02861	02924	01449	01449
0.040	0.001600	-0.003206	03206	0.03514×10^{-4}	03591	0.01607×10^{-2}	01607
.042	01764	03535	03535	04272	04366	01772	01772
.044	01936	- 03881	03881	05146	05260	01946	01946
.046	02116	- 04243	04243	06150	06284	02127	02128
.048	02304	- 04621	04621	07291	07452	02318	02318
0.050	0.002500	-0.005015	05015	0.08588×10^{-4}	08776	0.02516×10^{-2}	02517
.052	02704	- 05425	05426	01005×10^{-3}	01027	02723	02723
.054	02916	- 05852	05852	01169	01195	02938	02938
.056	03136	- 06295	06296	01352	01382	03161	03162
.058	03364	- 06755	06755	01557	01591	03393	03394
0.060	0.003600	-0.007231	07231	0.01783×10^{-3}	01822	0.03663×10^{-2}	03634
.062	03844	- 07723	07724	02034	02078	03882	03883
.064	04096	- 08232	08232	02310	02360	04139	04140
.066	04356	- 08757	08758	02612	02670	04404	04406
.068	04624	- 09299	09300	02945	03010	04679	04680
0.070	0.004900	-0.009857	09858	0.03308×10^{-3}	03381	0.04962×10^{-2}	04963
.072	05184	- 10432	10433	03704	03786	05253	05254
.074	05476	- 11023	11024	04135	04226	05553	05554
.076	05776	- 11632	11632	04602	04703	05862	05863
.078	06084	- 12256	12257	05108	05220	06179	06181

$$^1I_A = -\frac{2M^2\left(1 + \frac{\gamma-1}{2}M^2\right)}{1 - M^2}$$

$$^2I_f = \frac{\gamma M^4\left(1 + \frac{\gamma-1}{2}M^2\right)}{1 - M^2}$$

$$^3I_T = \frac{M^2(1 + \gamma M^2)\left(1 + \frac{\gamma-1}{2}M^2\right)}{1 - M^2}$$

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SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES

as those shown for $\gamma = 1.37$ unless an asterisk to left is increased one unit.]

^4I_R		^5I_c		$^6I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
182,482	178,572	0.00548x10^{-3}	00560	0.00074x10^{-3}	00080	0.002
45,621	44,643	00219x10^{-2}	00224	00296	00320	.004
20,276	19,841	00493	00504	00666	00720	.006
11,405	11,161	00877	00896	01184	01280	.008
7,299.4	7,143.0	0.00137x10^{-2}	00140	0.01850x10^{-3}	02000	0.010
5,069.1	4,960.5	00197	00202	02064	02880	.012
3,724.2	3,644.5	00268	00284	03626	03920	.014
2,851.4	2,790.3	00351	00358	04736	05120	.016
2,253.0	2,204.7	00444	00454	05994	06480	.018
1,825.0	1,785.9	0.00548x10^{-2}	00560	0.07399x10^{-3}	07999	0.020
1,508.2	1,475.9	00663	00678	08953	09679	.022
1,267.4	1,240.2	00789	00806	01066x10^{-2}	01152	.024
1,079.9	1,056.8	00926	00947	01250	01352	.026
931.16	911.22	01074	01098	01450	01568	.028
811.16	793.79	0.01233x10^{-2}	01260	0.01665x10^{-2}	01800	0.030
712.89	697.69	01403	01434	01894	02048	.032
631.45	618.04	01584	01619	02138	02311	.034
563.20	551.29	01776	01815	02397	02591	.036
505.70	494.80	01979	02022	02671	02887	.038
456.34	446.57	0.02193x10^{-2}	02241	0.02959x10^{-2}	03199	0.040
413.87	405.07	02418	02470	03262	03527	.042
377.21	369.09	02653	02711	03580	03870	.044
345.08	337.71	02900	02964	03913	04230	.046
316.99	310.16	03157	03227	04261	04606	.048
292.11	285.86	0.03426x10^{-2}	03502	0.04623x10^{-2}	04998	0.050
270.11	264.30	03706	03788	05000	05405	.052
250.45	245.10	03997	04085	05392	05829	.054
232.91	227.91	04298	04393	05798	06268	.056
217.10	212.48	04612	04713	06220	06723	.058
202.89	198.56	0.04935x10^{-2}	05044	0.06656x10^{-2}	07195	0.060
190.03	185.96	05270	05386	07106	07682	.062
178.32	174.53	05616	05739	07572	08185	.064
167.70	164.12	05973	06104	08052	08704	.066
157.99	154.62	06340	06480	08547	09239	.068
149.10	145.92	0.06719x10^{-2}	06867	0.09057x10^{-2}	09790	0.070
140.94	137.93	07109	07265	09581	10357	.072
133.43	130.58	07510	07675	10120	10940	.074
126.51	123.81	07921	08096	10674	11539	.076
120.11	117.55	08344	08528	11243	12153	.078

$$^4I_R = \frac{1 + \frac{\gamma-1}{2}M^2}{\gamma M^2}$$

$$^5I_c = \gamma M^2\left(1 + \frac{\gamma-1}{2}M^2\right)$$

$$^6I_{T,e} = \frac{\frac{\gamma-1}{2}M^2}{1 + \frac{\gamma-1}{2}M^2}$$

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IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	I_A		I_f		I_T	
		γ		γ		γ	
		1.37	1.40	1.37	1.40	1.37	1.40
0.080	0.006400	-0.012898	12899	0.05654×10^{-3}	05779	0.006505	06507
.082	06724	- 13556	13557	06244	06381	06840	06842
.084	07056	- 14231	14232	06878	07030	07184	07186
.086	07396	- 14923	14924	07560	07727	07537	07539
.088	07744	- 15631	15633	08292	08474	07899	07901
0.090	0.008100	-0.016357	16359	0.09076×10^{-3}	09275	0.008269	08272
.092	08464	- 17099	17101	09914	10132	08649	08652
.094	08836	- 17859	17861	10809	11047	09037	09041
.096	09216	- 18635	18638	11764	12024	09435	09439
.098	09604	- 19429	19432	12781	13063	09842	09846
0.100	0.010000	-0.020239	20242	0.13864×10^{-3}	14170	0.010258	10263
.102	10404	- 21068	21071	15014	15345	10684	10689
.104	10816	- 21912	21916	16235	16593	11118	11124
.106	11236	- 22774	22778	17526	17916	11562	11568
.108	11664	- 23654	23658	18900	19317	12016	12022
0.110	0.012100	-0.024551	24556	0.20349×10^{-3}	20799	0.012479	12486
.112	12544	- 25466	25470	21882	22365	12952	12959
.114	12996	- 26398	26403	23501	24019	13434	13442
.116	13456	- 27346	27352	25206	25764	13925	13934
.118	13924	- 28314	28320	27006	27603	14427	14436
0.120	0.014400	-0.029299	29305	0.28903×10^{-3}	29539	0.014938	14948
.122	14884	- 30300	30308	30892	31577	15459	15470
.124	15376	- 31322	31328	32990	33719	15991	16001
.126	15876	- 32358	32367	35189	35970	16531	16543
.128	16384	- 33414	33423	37500	38332	17082	17948
0.130	0.016900	-0.034489	34497	0.39920×10^{-3}	40810	0.017643	17657
.132	17424	- 35580	35590	42466	43408	18215	18229
.134	17956	- 36690	36700	45129	46129	18796	18811
.136	18496	- 37818	37829	47915	48974	19388	19404
.138	19044	- 38964	38975	50828	51957	19990	20007
0.140	0.019600	-0.040129	40140	0.53873×10^{-3}	55073	0.020603	20621
.142	20164	- 41312	41324	57062	58328	21227	21245
.144	20736	- 42512	42526	60384	61727	21860	21880
.146	21316	- 43732	43746	63855	65275	22505	22526
.148	21904	- 44970	44985	67473	68975	23160	23182
0.150	0.022500	-0.046227	46243	0.71236×10^{-3}	72833	0.023826	23850
.152	23104	- 47504	47519	75180	76852	24504	24528
.154	23716	- 48798	48815	79275	81038	25192	25218
.156	24336	- 50110	50129	83533	85395	25890	25918
.158	24964	- 51442	51462	87968	89929	26601	26630

SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for $\gamma = 1.37$ unless an asterisk to left is increased one unit.]

I_R		I_c		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
114.19	111.75	0.008778	08971	0.11826x10^-2	12784	0.080
108.69	106.37	09223	09426	12424	13430	.082
103.58	101.37	09680	09892	13037	14092	.084
98.822	96.720	10147	10370	13664	14770	.086
94.395	92.380	10624	10858	14306	15464	.088
90.250	88.326	0.011114	11358	0.14963x10^-2	16174	0.090
86.372	84.534	11614	11870	15634	16899	.092
82.746	80.981	12125	12392	16320	17641	.094
79.337	77.648	12648	12926	17021	18398	.096
76.140	74.517	13180	13471	17736	19171	.098
73.128	71.571	0.013725	14028	0.18466x10^-2	19960	0.100
70.296	68.798	14280	14596	19210	20765	.102
67.620	66.183	14848	15175	19970	21585	.104
65.100	63.714	15425	15766	20744	22422	.106
62.713	61.381	16014	16368	21532	23274	.108
60.460	59.175	0.016614	16981	0.22335x10^-2	24142	0.110
58.325	57.085	17225	17606	23153	25025	.112
56.299	55.105	17848	18242	23985	25925	.114
54.380	53.226	18481	18889	24832	26840	.116
52.557	51.442	19125	19548	25693	27771	.118
50.819	49.746	0.019780	20218	0.26569x10^-2	28717	0.120
49.176	48.133	20447	20900	27460	29680	.122
47.607	46.597	21125	21593	28365	30658	.124
46.112	45.134	21814	22297	29285	31652	.126
44.686	43.739	22514	23013	30219	32661	.128
43.332	42.408	0.023225	23740	0.31167x10^-2	33686	0.130
42.027	41.137	23948	24479	32131	34727	.132
40.785	39.923	24682	25287	33109	35784	.134
39.598	38.761	25427	25990	34101	36856	.136
38.464	37.650	26182	26763	35108	37944	.138
37.379	36.586	0.026949	27548	0.36129x10^-2	39047	0.140
36.334	35.567	27728	28343	37165	40166	.142
35.336	34.590	28517	29151	38215	41301	.144
34.378	33.652	29318	29970	39280	42451	.146
33.460	32.753	30130	30800	40359	43617	.148
32.581	31.889	0.030953	31642	0.41453x10^-2	44798	0.150
31.729	31.059	31787	32495	42560	45996	.152
30.913	30.261	32634	33360	43683	47208	.154
30.129	29.494	33490	34236	44820	48436	.156
29.374	28.756	34359	35124	45971	49680	.158

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TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	I_A γ 1.37	I_A γ 1.40	I_f γ 1.37	I_f γ 1.40	I_T γ 1.37	I_T γ 1.40
0.160	0.025600	-0.052794	52814	0.92575×10^{-3}	94643	0.027323	27354
.162	26244	- 54164	54186	97371	99543	28056	28088
.164	26896	- 55554	55576	10235×10^{-2}	10463	28801	28834
.166	27556	- 56962	56986	10752	10992	29556	29592
.168	28224	- 58390	58415	11289	11541	30324	30362
0.170	0.028900	-0.059838	59864	0.11846×10^{-2}	12110	0.031104	31143
.172	29584	- 61305	61333	12424	12701	31895	31936
.174	30276	- 62792	62821	13022	13314	32698	32742
.176	30976	- 64299	64328	13643	13948	33514	33559
.178	31684	- 65825	65856	14286	14606	34341	34389
0.180	0.032400	-0.067371	67404	0.14952×10^{-2}	15287	0.035181	35231
.182	33124	- 68937	68972	15642	15992	36033	36085
.184	33856	- 70524	70559	16355	16722	36898	36952
.186	34596	- 72130	72167	17094	17477	37774	37831
.188	35344	- 73757	73796	17857	18258	38664	38724
0.190	0.036100	-0.075404	75445	0.18646×10^{-2}	19065	0.039567	39629
.192	36864	- 77072	77114	19462	19899	40482	40547
.194	37636	- 78760	78804	20305	20761	41411	41478
.196	38416	- 80469	80515	21176	21652	42352	42423
.198	39204	- 82199	82247	22074	22571	43307	43381
0.200	0.040000	-0.083950	84000	0.23002×10^{-2}	23520	0.044275	44352
.202	40804	- 85722	85774	23960	24500	45257	45337
.204	41616	- 85715	87569	24948	25510	46252	46336
.206	42436	- 89329	89385	25967	26552	47261	47348
.208	43264	- 91165	91223	27018	27627	48284	48374
0.210	0.044100	-0.093022	93083	0.28100×10^{-2}	28735	0.049321	49415
.212	44944	- 94901	94964	29217	29876	50372	50470
.214	45796	- 96801	96867	30367	31053	51437	51539
.216	46656	- 98723	98792	31551	32265	52517	52622
.218	47524	- .100668	00739	32771	33513	53611	53721
0.220	0.048400	-0.10263	0271	0.34027×10^{-2}	34798	0.054720	54834
.222	49284	- 0462	0470	35320	36120	55844	55962
.224	50176	- 0663	0671	36651	37481	56982	57105
.226	51076	- 0867	0875	38020	38882	58136	58263
.228	51984	- 1072	1081	39428	40322	59305	59437
0.230	0.052900	-0.11280	1289	0.40876×10^{-2}	41804	0.060489	60626
.232	53824	- 1490	1500	42365	43327	61689	61831
.234	54756	- 1703	1712	43895	44893	62904	63052
.236	55696	- 1918	1928	45468	46502	64136	64288
.238	56644	- 2135	2145	47085	48156	65383	65541

SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for $\gamma = 1.37$ unless an asterisk to left is increased one unit.]

I_R		I_c		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
28.650	28.045	0.035238	36024	0.47137x10-2	50939	0.160
27.948	27.360	36129	36934	48317	52214	.162
27.274	26.700	37031	37857	49511	53504	.164
26.624	26.064	37944	38791	50720	54810	.166
25.997	25.451	38869	39737	51943	56131	.168
25.392	24.859	0.039805	40694	0.53181x10-2	57468	0.170
24.808	24.287	40752	41663	54432	58820	.172
24.244	23.735	41710	42643	55699	60188	.174
23.699	23.202	42680	43635	56979	61571	.176
23.173	22.687	43662	44639	58274	62969	.178
22.664	22.189	0.044654	45654	0.59583x10-2	64383	0.180
22.171	21.707	45658	46681	60906	65812	.182
21.695	21.241	46673	47719	62244	67257	.184
21.234	20.789	47700	48770	63596	68717	.186
20.787	20.352	48738	49831	64962	70192	.188
20.355	19.929	0.049787	50905	0.66342x10-2	71682	0.190
19.936	19.519	50848	51990	67736	73188	.192
19.529	19.122	51920	53087	69145	74710	.194
19.136	18.736	53004	54196	70568	76246	.196
18.754	18.363	54099	55316	72005	77798	.198
18.383	18.000	0.055206	56448	0.73456x10-2	79365	0.200
18.024	17.648	56324	57592	74922	80947	.202
17.675	17.307	57453	58747	76401	82545	.204
17.336	16.975	58594	59915	77895	84158	.206
17.006	16.653	59746	61094	79403	85786	.208
16.687	16.340	0.060910	62284	0.80925x10-2	87429	0.210
16.376	16.036	62085	63487	82461	89087	.212
16.074	15.740	63272	64702	84011	90761	.214
15.780	15.452	64470	65928	85575	92449	.216
15.494	15.173	65680	67166	87153	94153	.218
15.216	14.901	0.066902	68416	0.88745x10-2	95872	0.220
14.946	14.636	68135	69678	90352	97606	.222
14.682	14.378	69379	70951	91972	99355	.224
14.426	14.128	70635	72237	93606	*10112	.226
14.176	13.883	71903	73534	95254	*10290	.228
13.933	13.645	0.073182	74844	0.96916x10-2	*10469	0.230
13.696	13.414	74473	76165	98593	*10650	.232
13.466	13.188	75776	77498	10028x10-1	10833	.234
13.241	12.968	77090	78843	10199	11016	.236
13.021	12.753	78416	80200	10370	11202	.238

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TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for $\gamma = 1.40$ are same
appears, indicating that next digit

M	M^2	I_A		I_f		I_T	
		γ		γ		γ	
		1.37	1.40	1.37	1.40	1.37	1.40
0.240	0.057600	-0.12354	2365	0.48745×10^{-2}	49855	0.066646	66810
.242	58564	- 2576	2587	50451	51601	67926	68096
.244	59536	- 2800	2812	52203	53393	69222	69398
.246	60516	- 3027	3039	54002	55234	70535	70717
.248	61504	- 3256	3268	55848	57123	71865	72053
0.250	0.062500	-0.13488	3500	0.57743×10^{-2}	59062	0.073212	73406
.252	63504	- 3721	3734	59688	61053	74576	74777
.254	64516	- 3958	3971	61684	63095	75957	76165
.256	65536	- 4196	4210	63731	65190	77356	77570
.258	66564	- 4438	4452	65831	67339	78772	78994
0.260	0.067600	-0.14682	4696	0.67984×10^{-2}	69543	0.080206	80436
.262	68644	- 4928	4943	70192	71802	81658	81896
.264	69696	- 5177	5192	72456	74119	83129	83374
.266	70756	- 5428	5444	74776	76494	84618	84870
.268	71824	- 5682	5699	77155	78928	86126	86386
0.270	0.072900	-0.15939	5956	0.79592×10^{-2}	81422	0.087652	87921
.272	73984	- 6198	6215	82088	83978	89197	89475
.274	75076	- 6460	6478	84646	86596	90762	91048
.276	76176	- 6724	6743	87266	89277	92346	92641
.278	77284	- 6991	7010	89949	92024	93950	94254
0.280	0.078400	-0.17261	7281	0.92697	94836	0.095573	95887
.282	79524	- 7533	7554	95510	97716	97216	97540
.284	80656	- 7808	7830	98390	*10066	98880	99214
.286	81796	- 8086	8108	10134×10^{-1}	10368	.100564	00908
.288	82944	- 8367	8389	10435	10677	02269	02623
0.290	0.084100	-0.18650	8673	0.10744×10^{-1}	10993	0.10400	0436
.292	85264	- 8936	8960	11060	11316	0574	0612
.294	86436	- 9225	9250	11383	11647	0751	0790
.296	87616	- 9517	9542	11714	11986	0930	0970
.298	88804	- 9812	9838	12052	12332	1111	1152
0.300	0.090000	-0.20110	0136	0.12398×10^{-1}	12686	0.11295	1337
.302	91204	- 0410	0438	12751	13048	1480	1524
.304	92416	- 0713	0742	13113	13418	1668	1713
.306	93636	- 1020	1049	13482	13796	1858	1904
.308	94864	- 1329	1359	13860	14183	2051	2098
0.310	0.096100	-0.21641	1672	0.14246×10^{-1}	14579	0.12245	2294
.312	97344	- 1957	1988	14641	14983	2442	2492
.314	98596	- 2275	2307	15044	15396	2642	2693
.316	99856	- 2597	2630	15456	15818	2844	2897
.318	.101124	- 2921	2955	15877	16249	3048	3102

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SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for $\gamma = 1.37$ unless an asterisk to left is increased one unit.]

I_R		I_e		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
12.807	12.544	0.079753	81569	0.010544	11389	0.240
12.599	12.340	81102	82950	10718	11577	.242
12.395	12.140	82463	84343	10894	11767	.244
12.197	11.946	83835	85748	11072	11958	.246
12.003	11.756	85219	87165	11250	12151	.248
11.814	11.571	0.086615	88594	0.011430	12346	0.250
11.629	11.391	88023	90035	11612	12542	.252
11.449	11.214	89442	91488	11795	12739	.254
11.273	11.042	90873	92953	11979	12938	.256
11.101	10.874	92316	94430	12164	13138	.258
10.933	10.709	0.093770	95920	0.012352	13340	0.260
10.768	10.548	95236	97421	12540	13543	.262
10.608	10.391	96715	98934	12730	13748	.264
10.451	10.238	98205	*00460	12921	13954	.266
10.298	10.088	99706	*01998	13113	14161	.268
10.148	9.9410	0.10122	0355	0.013307	14370	0.270
10.001	9.7974	0274	0511	13502	14581	.272
9.8576	9.6571	0428	0668	13699	14793	.274
9.7172	9.5197	0583	0827	13897	15007	.276
9.5798	9.3852	0739	0987	14096	15222	.278
9.4453	9.2536	0.10897	1148	0.014297	15438	0.280
9.3137	9.1248	1055	1310	14499	15656	.282
9.1849	8.9988	1214	1474	14702	15875	.284
9.0587	8.8754	1376	1639	14907	16096	.286
8.9353	8.7545	1538	1805	15113	16318	.288
8.8143	8.6362	0.11701	1972	0.015320	16542	0.290
8.6958	8.5202	1866	2141	15529	16767	.292
8.5798	8.4066	2031	2310	15739	16993	.294
8.4660	8.2953	2198	2481	15950	17221	.296
8.3546	8.1862	2366	2653	16163	17451	.298
8.2453	8.0794	0.12535	2827	0.016377	17682	0.300
8.1383	7.9746	2706	3002	16593	17914	.302
8.0333	7.8719	2878	3177	16810	18148	.304
7.9304	7.7712	3050	3354	17028	18383	.306
7.8295	7.6724	3224	3533	17247	18620	.308
7.7305	7.5756	0.13400	3713	0.017468	18858	0.310
7.6335	7.4806	3576	3894	17690	19097	.312
7.5382	7.3874	3754	4076	17914	19338	.314
7.4448	7.2960	3933	4259	18138	19580	.316
7.3532	7.2063	4113	4444	18364	19824	.318

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TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same appears, indicating that next digit

M	M^2	I_A		I_f		I_T	
		γ		γ		γ	
		1.37	1.40	1.37	1.40	1.37	1.40
0.320	0.10240	-0.23249	3284	0.016308	16690	0.13255	3311
.322	0368	- 3579	3615	16747	17140	3464	3522
.324	0498	- 3913	3950	17196	17599	3676	3735
.326	0628	- 4250	4288	17654	18069	3891	3951
.328	0758	- 4591	4630	18122	18548	4108	4170
0.330	0.10890	-0.24934	4974	0.018600	19038	0.14327	4391
.332	1022	- 5281	5322	19088	19538	4549	4615
.334	1156	- 5631	5673	19586	20048	4774	4841
.336	1290	- 5984	6027	20095	20569	5002	5071
.338	1424	- 6341	6385	20614	21101	5232	5303
0.340	0.11560	-0.26701	6746	0.021144	21643	0.15465	5538
.342	1696	- 7065	7111	21684	22197	5701	5775
.344	1834	- 7431	7479	22236	22762	5939	6016
.346	1972	- 7802	7851	22799	23339	6181	6259
.348	2110	- 8176	8226	23373	23928	6425	6506
0.350	0.12250	-0.28553	8604	0.023960	24528	0.16672	6755
.352	2390	- 8934	8986	24557	25141	6923	7007
.354	2532	- 9318	9372	25167	25766	7176	7263
.356	2674	- 9706	9762	25789	26403	7432	7521
.358	2816	- .30098	0155	26424	27053	7691	7783
0.360	0.12960	-0.30493	0551	0.027071	27716	0.17954	8047
.362	3104	- 0892	0952	27731	28392	8219	8315
.364	3250	- 1295	1356	28404	29082	8488	8586
.366	3396	- 1702	1764	29090	29785	8760	8860
.368	3542	- 2112	2176	29789	30502	9035	9138
0.370	0.13690	-0.32526	2591	0.030502	31232	0.19313	9419
.372	3838	- 2944	3011	31229	31977	9595	9703
.374	3988	- 3366	3434	31970	32737	9880	9991
.376	4138	- 3792	3862	32725	33511	.20169	0282
.378	4288	- 4222	4293	33495	34300	0460	0577
0.380	0.14440	-0.34656	4729	0.034279	35104	0.20756	0875
.382	4592	- 5094	5168	35079	35923	1055	1176
.384	4746	- 5536	5612	35894	36759	1357	1482
.386	4900	- 5982	6060	36724	37609	1663	1791
.388	5054	- 6432	6512	37570	38477	1973	2104
0.390	0.15210	-0.36886	6968	0.038431	39360	0.22286	2420
.392	5366	- 7345	7429	39309	40260	2603	2740
.394	5524	- 7808	7894	40204	41177	2924	3064
.396	5682	- 8275	8363	41115	42111	3249	3392
.398	5840	- 8747	8836	42043	43063	3578	3724

NACA

SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for γ = 1.37 unless an asterisk to left is increased one unit.]

I_R		I_c		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
7.2632	7.1183	0.14295	4630	0.018592	20069	0.320
7.1750	7.0319	4477	4817	18820	20316	.322
7.0883	6.9472	4661	5005	19051	20563	.324
7.0033	6.8639	4846	5195	19282	20813	.326
6.9198	6.7822	5032	5386	19515	21064	.328
6.8378	6.7020	0.15220	5578	0.019749	21316	0.330
6.7572	6.6232	5409	5772	19984	21569	.332
6.6782	6.5458	5599	5966	20220	21824	.334
6.6005	6.4698	5790	6162	20458	22081	.336
6.5242	6.3951	5982	6360	20698	22338	.338
6.4493	6.3218	0.16176	6558	0.020938	22598	0.340
6.3756	6.2497	6371	6758	21180	22858	.342
6.3033	6.1790	6567	6959	21423	23120	.344
6.2322	6.1094	6764	7162	21668	23383	.346
6.1623	6.0410	6963	7365	21913	23648	.348
6.0936	5.9738	0.17163	7570	0.022160	23914	0.350
6.0261	5.9077	7364	7776	22409	24182	.352
5.9597	5.8428	7566	7984	22658	24450	.354
5.8945	5.7789	7770	8193	22909	24721	.356
5.8303	5.7161	7975	8403	23161	24992	.358
5.7672	5.6543	0.18181	8614	0.023415	25265	0.360
5.7051	5.5936	8388	8827	23669	25539	.362
5.6441	5.5339	8597	9041	23925	25815	.364
5.5840	5.4751	8807	9256	24183	26092	.366
5.5250	5.4173	9018	9473	24441	26371	.368
5.4669	5.3604	0.19230	9691	0.024701	26650	0.370
5.4097	5.3045	9444	9910	24962	26931	.372
5.3534	5.2494	9659	*0130	25224	27214	.374
5.2981	5.1952	9875	*0352	25488	27498	.376
5.2436	5.1419	.20092	0575	25753	27783	.378
5.1899	5.0894	0.20311	0800	0.026019	28069	0.380
5.1371	5.0378	0531	1026	26286	28357	.382
5.0852	4.9869	0753	1253	26555	28646	.384
5.0340	4.9369	0975	1481	26825	28937	.386
4.9836	4.8875	1199	1711	27096	29229	.388
4.9340	4.8390	0.21424	1942	0.027368	29522	0.390
4.8852	4.7912	1650	2174	27642	29816	.392
4.8371	4.7442	1878	2408	27917	30112	.394
4.7897	4.6978	2107	2643	28193	30409	.396
4.7431	4.6521	2337	2879	28470	30708	.398

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TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	I_A γ 1.37	I_A γ 1.40	I_f γ 1.37	I_f γ 1.40	I_T γ 1.37	I_T γ 1.40
0.400	0.16000	-0.39223	9314	0.042988	44032	0.23910	4060
.402	6160	- 9703	9797	43951	45019	4247	4400
.404	6322	- .40188	0284	44932	46025	4587	4744
.406	6484	- 0678	0775	45930	47048	4932	5092
.408	6646	- 1172	1271	46947	48091	5281	5445
0.410	0.16810	-0.41670	1772	0.047983	49153	0.25634	5801
.412	6974	- 2174	2278	49037	50235	5990	6162
.414	7140	- 2682	2788	50111	51336	6352	6528
.416	7306	- 3194	3303	51204	52457	6718	6897
.418	7472	- 3712	3823	52317	53598	7088	7271
0.420	0.17640	-0.44234	4348	0.053450	54760	0.27462	7650
.422	7808	- 4762	4877	54604	55943	7841	8033
.424	7978	- 5294	5412	55778	57148	8225	8421
.426	8148	- 5831	5952	56973	58374	8613	8813
.428	8318	- 6373	6496	58190	59622	9006	9210
0.430	0.18490	-0.46921	7046	0.059428	60892	0.29403	9612
.432	8662	- 7473	7602	60688	62185	.9805	*0019
.434	8836	- 8031	8162	61971	63501	.30212	0431
.436	9010	- 8594	8728	63277	64840	0625	0848
.438	9184	- 9162	9299	64605	66204	1042	1270
0.440	0.19360	-0.49736	9875	0.065957	67591	0.31464	1697
.442	9536	- .50315	0457	67333	69002	1891	2129
.444	9714	- 0899	1044	68733	70439	2323	2566
.446	9892	- 1489	1637	70158	71900	2760	3009
.448	.20070	- 2085	2236	71607	73388	3203	3457
0.450	0.20250	-0.52686	2840	0.073082	74901	0.33651	3910
.452	0430	- 3293	3451	74583	76441	4105	4369
.454	0612	- 3906	4067	76110	78008	4564	4834
.456	0794	- 4525	4688	77663	79602	5029	5304
.458	0976	- 5149	5316	79243	81224	5499	5780
0.460	0.21160	-0.55780	5950	0.080850	82873	0.35975	6262
.462	1344	- 6416	6590	82486	84552	6457	6750
.464	1530	- 7059	7236	84149	86259	6944	7244
.466	1716	- 7708	7888	85841	87995	7438	7744
.468	1902	- 8363	8547	87562	89762	7938	8250
0.470	0.22090	-0.59024	9212	0.089313	91559	0.38443	8762
.472	2278	- 9692	9883	91094	93387	8955	9280
.474	2468	- .60366	0561	92904	95246	9473	9805
.476	2658	- 1046	1245	94747	97137	9998	*0336
.478	2848	- 1734	1937	96620	99061	.40529	0874

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SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for γ = 1.37 unless an asterisk to left is increased one unit.]

I_R		I_c		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
4.6971	4.6071	0.22569	3117	0.028749	31008	0.400
4.6518	4.5628	2802	3356	29029	31309	.402
4.6072	4.5192	3036	3596	29310	31611	.404
4.5632	4.4762	3271	3838	29592	31915	.406
4.5199	4.4338	3508	4081	29876	32220	.408
4.4773	4.3920	0.23746	4325	0.030160	32526	0.410
4.4352	4.3509	3985	4571	30446	32834	.412
4.3937	4.3103	4226	4818	30734	33143	.414
4.3529	4.2703	4468	5066	31022	33453	.416
4.3126	4.2309	4711	5316	31312	33765	.418
4.2729	4.1921	0.24956	5567	0.031603	34078	0.420
4.2338	4.1538	5201	5820	31895	34392	.422
4.1952	4.1161	5448	6074	32188	34708	.424
4.1572	4.0788	5697	6329	32482	35024	.426
4.1197	4.0421	5947	6585	32778	35342	.428
4.0827	4.0059	0.26198	6843	0.033075	35661	0.430
4.0462	3.9703	6450	7103	33373	35982	.432
4.0103	3.9351	6704	7363	33672	36304	.434
3.9748	3.9004	6959	7625	33973	36627	.436
3.9398	3.8661	7215	7889	34275	36951	.438
3.9053	3.8323	0.27473	8153	0.034578	37277	0.440
3.8713	3.7990	7732	8420	34882	37604	.442
3.8377	3.7662	7993	8687	35187	37932	.444
3.8046	3.7338	8254	8956	35493	38261	.446
3.7719	3.7018	8517	9226	35801	38592	.448
3.7396	3.6702	0.28782	9498	0.036110	38924	0.450
3.7078	3.6390	9048	9771	36420	39251	.452
3.6764	3.6083	9315	*0046	36731	39591	.454
3.6454	3.5780	9583	*0322	37043	39927	.456
3.6148	3.5480	9853	*0599	37357	40264	.458
3.5846	3.5185	0.30124	0878	0.037671	40602	0.460
3.5548	3.4893	0397	1158	37987	40941	.462
3.5254	3.4606	0670	1439	38304	41282	.464
3.4963	3.4321	0946	1722	38622	41623	.466
3.4677	3.4041	1222	2007	38942	41966	.468
3.4394	3.3764	0.31500	2292	0.039262	42311	0.470
3.4114	3.3490	1779	2580	39584	42656	.472
3.3838	3.3220	2060	2868	39906	43003	.474
3.3566	3.2954	2342	3158	40230	43351	.476
3.3297	3.2690	2625	3450	40555	43700	.478

TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	I_A, γ = 1.37	I_A, γ = 1.40	I_f, γ = 1.37	I_f, γ = 1.40	I_T, γ = 1.37	I_T, γ = 1.40
0.480	0.23040	-0.62427	2634	0.098525	*01017	0.41066	1419
.482	3232	- 3128	3339	.100463	03006	1610	1970
.484	3426	- 3835	4050	02434	05029	2161	2528
.486	3620	- 4550	4769	04438	07087	2719	3093
.488	3814	- 5271	5494	06476	09180	3283	3665
0.490	0.24010	-0.65999	6227	0.10855	1131	0.43854	4244
.492	4206	- 6735	6967	1066	1347	4433	4831
.494	4404	- 7478	7714	1280	1567	5019	5424
.496	4602	- 8228	8468	1498	1791	5612	6025
.498	4800	- 8985	9230	1719	2019	6212	6634
0.500	0.25000	-0.69750	*0000	0.11945	2250	0.46820	7250
.502	5200	- .70522	0777	2174	2485	7435	7874
.504	5402	- 1303	1562	2407	2724	8058	8506
.506	5604	- 2090	2355	2644	2968	8689	9145
.508	5806	- 2886	3155	2884	3215	9328	9793
0.510	0.26010	-0.73690	3964	0.13129	3467	0.49974	*0449
.512	6214	- 4502	4781	3378	3722	.50629	1113
.514	6420	- 5321	5606	3631	3982	1292	1785
.516	6626	- 6149	6439	3888	4247	1963	2466
.518	6832	- 6986	7281	4150	4516	2643	3156
0.520	0.27040	-0.77831	8131	0.14416	4789	0.53332	3854
.522	7248	- 8684	8990	4686	5066	4029	4562
.524	7458	- 9546	9858	4961	5349	4734	5278
.526	7668	- .80417	0734	5241	5636	5449	6003
.528	7878	- 1297	1620	5525	5928	6173	6738
0.530	0.28090	-0.82185	2515	0.15814	6225	0.56906	7482
.532	8302	- 3083	3418	6107	6527	7649	8236
.534	8516	- 3990	4331	6406	6833	8401	8999
.536	8730	- 4906	5254	6709	7145	9163	9772
.538	8944	- 5832	6186	7018	7462	9934	*0555
0.540	0.29160	-0.86768	7128	0.17332	7784	0.60715	1348
.542	9376	- 7713	8079	7650	8112	1507	2152
.544	9594	- 8667	9041	7974	8445	2308	2966
.546	9812	- 9632	*0012	8304	8784	3120	3790
.548	.30030	- .90607	0994	8639	9128	3942	4625
0.550	0.30250	-0.91592	1986	0.18979	9478	0.64775	5471
.552	0470	- 2588	2989	9325	9834	5619	6328
.554	0692	- 3594	4002	9677	*0195	6474	7196
.556	0914	- 4611	5026	.20035	0563	7340	8076
.558	1136	- 5638	9606	0398	0937	8217	8967

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SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for γ = 1.37 unless an asterisk to left is increased one unit.]

I_R γ 1.37	I_R γ 1.40	I_c γ 1.37	I_c γ 1.40	$I_{T,e}$ γ 1.37	$I_{T,e}$ γ 1.40	M
3.3031	3.2431	0.32910	3742	0.040882	44050	0.480
3.2769	3.2174	3196	4037	41209	44402	.482
3.2510	3.1920	3484	4332	41537	44754	.484
3.2254	3.1670	3773	4630	41867	45108	.486
3.2001	3.1422	4063	4928	42198	45463	.488
3.1751	3.1178	0.34355	5228	0.042529	45820	0.490
3.1505	3.0937	4648	5530	42862	46177	.492
3.1261	3.0698	4942	5832	43196	46536	.494
3.1020	3.0463	5238	6137	43532	46896	.496
3.0782	3.0230	5535	6443	43868	47257	.498
3.0547	3.0000	0.35834	6750	0.044206	47619	0.500
3.0315	2.9773	6134	7059	44544	47982	.502
3.0086	2.9548	6436	7369	44884	48347	.504
2.9859	2.9326	6738	7680	45224	48713	.506
2.9635	2.9107	7043	7994	45566	49080	.508
2.9414	2.8891	0.37348	8308	0.045909	49448	0.510
2.9195	2.8676	7655	8624	46254	49817	.512
2.8979	2.8465	7964	8942	46599	50187	.514
2.8765	2.8256	8274	9261	46945	50559	.516
2.8554	2.8049	8585	9581	47292	50932	.518
2.8345	2.7844	0.38898	9903	0.047641	51305	0.520
2.8138	2.7642	9212	*0227	47990	51680	.522
2.7934	2.7443	9528	*0552	48341	52057	.524
2.7732	2.7245	9845	*0878	48693	52434	.526
2.7533	2.7050	.40163	1206	49046	52812	.528
2.7336	2.6857	0.40483	1535	0.049399	53192	0.530
2.7141	2.6666	0804	1866	49754	53572	.532
2.6948	2.6478	1127	2199	50110	53954	.534
2.6757	2.6291	1452	2532	50467	54337	.536
2.6569	2.6106	1777	2868	50826	54721	.538
2.6382	2.5924	0.42104	3205	0.051185	55106	0.540
2.6198	2.5744	2433	3543	51545	55492	.542
2.6015	2.5565	2763	3883	51906	55880	.544
2.5835	2.5389	3094	4225	52269	56268	.546
2.5657	2.5214	3427	4568	52632	56658	.548
2.5480	2.5041	0.43762	4912	0.052997	57049	0.550
2.5306	2.4870	4098	5258	53362	57440	.552
2.5133	2.4702	4435	5606	53729	57833	.554
2.4962	2.4534	4774	5955	54096	58227	.556
2.4793	2.4369	5114	6306	54465	58622	.558

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TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	I_A		I_f		I_T	
		γ		γ		γ	
		1.37	1.40	1.37	1.40	1.37	1.40
0.560	0.31360	-0.96677	7106	0.20768	1317	0.69106	9870
.562	1584	- 7726	8163	1143	1703	.70006	0785
.564	1810	- 8787	9232	1525	2096	0919	1712
.566	2036	- 9859	*0312	1913	2495	1843	2651
.568	2262	-1.00942	1403	2308	2901	2779	3602
0.570	0.32490	-1.0204	251	0.22709	3313	0.73728	4566
.572	2718	- 314	362	3117	3732	4690	5544
.574	2948	- 426	475	3532	4159	5664	6534
.576	3178	- 540	589	3953	4592	6651	7537
.578	3408	- 654	704	4381	5033	7651	8554
0.580	0.33640	-1.0770	821	0.24817	5481	0.78665	9584
.582	3872	- 886	939	5260	5936	9692	*0629
.584	4106	- .1005	058	5710	6399	.80733	1687
.586	4340	- 124	178	6167	6870	1788	2760
.588	4574	- 245	300	6632	7348	2858	3848
0.590	0.34810	-1.1367	423	0.27105	7835	0.83942	4950
.592	5046	- 491	548	7586	8329	5040	6067
.594	5284	- 616	674	8075	8832	6154	7200
.596	5522	- 742	801	8572	9343	7282	8348
.598	5760	- 870	930	9077	9863	8427	9511
0.600	0.36000	-1.1999	*060	0.29590	*0391	0.89586	*0691
.602	6240	- .2130	192	.30112	0928	.90762	1887
.604	6482	- 262	325	0643	1475	1954	3100
.606	6724	- 396	460	1183	2030	3163	4329
.608	6966	- 531	596	1731	2595	4385	5576
0.610	0.37210	-1.2668	734	0.32290	3169	0.95630	6840
.612	7454	- 806	874	2857	3753	6890	8122
.614	7700	- 947	*015	3434	4346	8167	9422
.616	7946	- .3088	158	4020	4950	9462	*0740
.618	8192	- 232	302	4616	5564	1.00775	2076
0.620	0.38440	-1.3377	449	0.35223	6188	1.0211	343
.622	8688	- 524	597	5840	6823	346	481
.624	8938	- 672	747	6466	7468	483	620
.626	9188	- 822	898	7104	8124	622	761
.628	9438	- 974	*052	7752	8792	762	905
0.630	0.39690	-1.4128	207	0.38412	9471	1.0905	*050
.632	9942	- 284	364	9082	*0161	.1050	198
.634	.40196	- 442	523	9764	*0863	197	348
.636	0450	- 602	684	.40458	1577	347	500
.638	0704	- 763	847	1163	2304	498	654

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SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for γ = 1.37 unless an asterisk to left is increased one unit.]

I_R		I_c		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
2.4626	2.4206	0.45456	6658	0.054835	59018	0.560
2.4461	2.4044	5799	7011	55205	59416	.562
2.4297	2.3884	6144	7367	55577	59814	.564
2.4135	2.3725	6490	7723	55950	60213	.566
2.3975	2.3568	6838	8082	56324	60614	.568
2.3817	2.3413	0.47187	8442	0.056698	61015	0.570
2.3660	2.3260	7537	8803	57074	61418	.572
2.3504	2.3108	7890	9166	57451	61821	.574
2.3351	2.2958	8243	9531	57829	62226	.576
2.3199	2.2809	8598	9897	58208	62632	.578
2.3049	2.2662	0.48955	*0265	0.058588	63039	0.580
2.2900	2.2516	9313	*0634	58969	63447	.582
2.2752	2.2372	9673	*1005	59351	63856	.584
2.2606	2.2229	.50034	1377	59734	64266	.586
2.2462	2.2088	0397	1751	60117	64676	.588
2.2319	2.1948	0.50761	2127	0.060502	65089	0.590
2.2178	2.1810	1127	2504	60888	65502	.592
2.2038	2.1673	1494	2883	61275	65916	.594
2.1899	2.1537	1863	3263	61663	66331	.596
2.1762	2.1403	2233	3645	62052	66747	.598
2.1626	2.1270	0.52605	4029	0.062441	67164	0.600
2.1492	2.1138	2978	4414	62832	67582	.602
2.1358	2.1008	3353	4801	63224	68002	.604
2.1227	2.0879	3729	5189	63617	68422	.606
2.1096	2.0751	4107	5579	64010	68843	.608
2.0967	2.0625	0.54487	5971	0.064405	69265	0.610
2.0839	2.0499	4868	6364	64800	69689	.612
2.0712	2.0375	5251	6759	65197	70113	.614
2.0586	2.0253	5635	7155	65595	70538	.616
2.0462	2.0131	6021	7554	65993	70964	.618
2.0339	2.0010	0.56408	7953	0.066393	71391	0.620
2.0217	1.9891	6797	8355	66793	71820	.622
2.0096	1.9773	7187	8758	67194	72249	.624
1.9977	1.9656	7579	9162	67596	72679	.626
1.9858	1.9540	7973	9569	68000	73110	.628
1.9741	1.9425	0.58368	9977	0.068404	73542	0.630
1.9625	1.9311	8765	*0386	68809	73975	.632
1.9510	1.9199	9163	*0798	69215	74409	.634
1.9396	1.9087	9563	*1211	69622	74844	.636
1.9283	1.8977	9964	*1625	70030	75280	.638

NACA

TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	I_A, γ = 1.37	I_A, γ = 1.40	I_f, γ = 1.37	I_f, γ = 1.40	I_T, γ = 1.37	I_T, γ = 1.40
0.640	0.40960	-1.4927	*012	0.41881	3042	1.1652	810
.642	1216	- .5092	179	2611	3794	807	969
.644	1474	- 260	348	3353	4558	965	*130
.646	1732	- 430	519	4108	5336	.2126	293
.648	1990	- 602	693	4876	6126	288	459
0.650	0.42250	-1.5776	868	0.45657	6931	1.2454	627
.652	2510	- 952	*046	6452	7749	621	798
.654	2772	- .6130	226	7260	8582	791	971
.656	3034	- 311	409	8082	9429	964	*147
.658	3296	- 494	594	8919	*0291	.3139	326
0.660	0.43560	-1.6680	781	0.49770	*1168	1.3317	507
.662	3824	- 868	970	.50636	2060	497	691
.664	4090	- .7058	162	1517	2967	681	878
.666	4356	- 251	357	2414	3891	867	*068
.668	4622	- 446	554	3326	4831	.4056	260
0.670	0.44890	-1.7644	754	0.54255	5787	1.4248	456
.672	5158	- 844	956	5199	6761	442	654
.674	5428	- .8048	161	6161	7751	640	856
.676	5698	- 254	369	7139	8759	841	*060
.678	5968	- 462	580	8135	9786	.5045	268
0.680	0.46240	-1.8674	793	0.59149	*0830	1.5252	480
.682	6512	- 888	*010	.60180	1893	462	694
.684	6786	- .9106	229	1230	2975	676	912
.686	7060	- 326	452	2299	4077	893	*134
.688	7334	- 550	677	3388	5199	.6114	358
0.690	0.47610	-1.9776	906	0.64495	6340	1.6338	587
.692	7886	-2.0006	138	5623	7503	565	819
.694	8164	- 239	373	6772	8686	797	*055
.696	8442	- 475	611	7941	9892	.7032	295
.698	8720	- 715	853	9132	*1119	270	539
0.700	0.49000	-2.0958	*099	0.70344	2369	1.7513	786
.702	9280	- .1204	348	1579	3642	760	*038
.704	9562	- 454	600	2836	4938	.8011	294
.706	9844	- 708	857	4117	6259	266	554
.708	.50126	- 965	*117	5422	7604	525	819
0.710	0.50410	-2.2227	380	0.76751	8974	1.8788	*088
.712	0694	- 492	648	8104	*0370	.9056	361
.714	0980	- 761	920	9484	*1792	329	639
.716	1266	- .3034	196	.80889	3241	606	922
.718	1552	- 311	476	2320	4717	888	*210

NACA

SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for γ = 1.37 unless an asterisk to left is increased one unit.]

I_R		I_c		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
1.9171	1.8867	0.60367	2042	0.070438	75717	0.640
1.9060	1.8759	0772	2460	70848	76155	.642
1.8950	1.8651	1178	2879	71259	76594	.644
1.8841	1.8545	1586	3300	71670	77034	.646
1.8734	1.8439	1996	3724	72083	77474	.648
1.8627	1.8335	0.62407	4148	0.072496	77716	0.650
1.8521	1.8231	2819	4575	72910	78359	.652
1.8416	1.8129	3234	5002	73325	78802	.654
1.8312	1.8027	3650	5432	73742	79247	.656
1.8209	1.7926	4067	5864	74158	79692	.658
1.8107	1.7826	0.64486	6297	0.074576	80138	0.660
1.8006	1.7727	4907	6732	74995	80586	.662
1.7906	1.7629	5330	7168	75414	81034	.664
1.7807	1.7532	5754	7607	75835	81483	.666
1.7708	1.7436	6179	8047	76256	81933	.668
1.7611	1.7340	0.66607	8488	0.076679	82384	0.670
1.7514	1.7246	7036	8932	77102	82835	.672
1.7418	1.7152	7466	9377	77526	83288	.674
1.7323	1.7059	7898	9824	77950	83742	.676
1.7229	1.6967	8332	*0272	78376	84196	.678
1.7136	1.6876	0.68768	*0723	0.078803	84651	0.680
1.7044	1.6785	9205	*1175	79230	85108	.682
1.6952	1.6696	9644	*1629	79659	85565	.684
1.6861	1.6607	.70085	2084	80088	86023	.686
1.6771	1.6519	0527	2542	80518	86482	.688
1.6682	1.6431	0.70971	3001	0.080949	86941	0.690
1.6593	1.6345	1416	3462	81380	87402	.692
1.6506	1.6259	1864	3924	81813	87864	.694
1.6418	1.6174	2312	4389	82246	88326	.696
1.6332	1.6089	2763	4855	82680	88789	.698
1.6247	1.6006	0.73215	5323	0.083116	89253	0.700
1.6162	1.5923	3669	5793	83551	89718	.702
1.6078	1.5841	4125	6264	83988	90184	.704
1.5995	1.5759	4582	6737	84426	90651	.706
1.5912	1.5678	5042	7212	84864	91118	.708
1.5830	1.5598	0.75502	7689	0.085303	91586	0.710
1.5749	1.5519	5965	8168	85743	92055	.712
1.5668	1.5440	6429	8648	86184	92525	.714
1.5588	1.5362	6895	9131	86626	92996	.716
1.5509	1.5284	7363	9615	87068	93468	.718

NACA

TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	I_A		I_f		I_T	
		γ		γ		γ	
		1.37	1.40	1.37	1.40	1.37	1.40
0.720	0.51840	-2.3593	760	0.83779	6221	2.0174	502
.722	2128	- 879	*049	5266	7755	466	800
.724	2418	- .4169	342	6781	9317	762	*103
.726	2708	- 464	640	8325	*0909	.1064	411
.728	2998	- 763	942	9899	*2533	371	724
0.730	0.53290	-2.5067	249	0.91503	4187	2.1684	*043
.732	3582	- 376	561	3138	5874	.2002	368
.734	3876	- 689	878	4806	7594	325	698
.736	4170	- .6008	200	6506	9348	655	*035
.738	4464	- 332	527	8240	*1136	990	*377
0.740	0.54760	-2.6661	860	1.0001	296	2.3331	726
.742	5056	- 996	*198	181	482	679	*081
.744	5354	- .7336	542	365	672	.4033	442
.746	5652	- 681	891	552	865	393	811
.748	5950	- .8033	246	744	*063	760	*186
0.750	0.56250	-2.8390	607	1.0939	*264	2.5134	568
.752	6550	- 754	974	.1138	470	515	957
.754	6852	- .9123	348	342	679	903	*353
.756	7154	- 499	728	549	893	.6299	757
.758	7456	- 882	*114	761	*112	702	*169
0.760	0.57760	-3.0271	508	1.1977	*335	2.7112	589
.762	8064	- 667	908	.2197	563	531	8017
.764	8370	- .1070	315	423	795	958	*453
.766	8676	- 480	730	653	*032	.8393	897
.768	8982	- 898	*152	888	*275	837	*351
0.770	0.59290	-3.2323	582	1.3128	522	2.9289	814
.772	9598	- 756	*020	373	775	751	*285
.774	9908	- .3197	465	623	*034	3.0221	766
.776	.60218	- 646	919	879	*298	702	*258
.778	0528	- .4104	382	.4140	568	.1192	759
0.780	0.60840	-3.4570	853	1.4407	843	3.1692	*270
.782	1152	- .5045	334	680	*125	.2203	792
.784	1466	- 529	823	959	*413	724	*325
.786	1780	- .6023	323	.5245	708	.3256	869
.788	2094	- 526	831	536	*009	799	*425
0.790	0.62410	-3.7040	350	1.5835	*317	3.4354	992
.792	2726	- 563	880	.6140	632	921	*572
.794	3044	- .8097	420	452	955	.5501	*165
.796	3362	- 642	971	772	*285	.6092	770
.798	3680	- .9198	533	.7098	622	697	*389

NACA

SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for γ = 1.37 unless an asterisk to left is increased one unit.]

I_R		I_c		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
1.5431	1.5207	0.77832	*0101	0.087511	93940	0.720
1.5353	1.5131	8303	*0588	87955	94414	.722
1.5276	1.5055	8776	*1078	88400	94888	.724
1.5199	1.4980	9250	*1569	88846	95363	.726
1.5123	1.4906	9727	*2062	89292	95838	.728
1.5048	1.4832	0.80205	2558	0.089739	96315	0.730
1.4973	1.4759	0685	3054	90187	96792	.732
1.4899	1.4687	1166	3553	90636	97270	.734
1.4825	1.4615	1650	4054	91086	97749	.736
1.4752	1.4543	2134	4556	91536	98229	.738
1.4680	1.4472	0.82621	5060	0.091987	98709	0.740
1.4608	1.4402	3110	5566	92439	99191	.742
1.4537	1.4333	3600	6074	92892	99673	.744
1.4466	1.4264	4092	6584	93345	*00156	.746
1.4396	1.4195	4586	7096	93799	*00639	.748
1.4327	1.4127	0.85082	7609	0.094254	*01124	0.750
1.4258	1.4060	5579	8125	94710	*01609	.752
1.4190	1.3993	6078	8642	95166	*02095	.754
1.4122	1.3926	6579	9161	95624	*02581	.756
1.4054	1.3860	7082	9682	96082	*03069	.758
1.3988	1.3795	0.87587	*0205	0.096540	*03557	0.760
1.3921	1.3730	8093	*0730	97000	*04046	.762
1.3856	1.3666	8602	*1257	97460	*04536	.764
1.3790	1.3602	9112	*1786	97921	*05026	.766
1.3726	1.3539	9623	*2316	98382	*05517	.768
1.3662	1.3476	0.90137	2849	0.098844	*06009	0.770
1.3598	1.3414	0652	3383	99308	*06502	.772
1.3535	1.3352	1170	3920	99772	*06996	.774
1.3472	1.3290	1689	4458	.100236	07490	.776
1.3410	1.3229	2210	4998	00701	07985	.778
1.3348	1.3169	0.92732	5540	0.10117	0848	0.780
1.3287	1.3109	3257	6084	0163	0898	.782
1.3226	1.3050	3783	6630	0210	0947	.784
1.3165	1.2990	4312	7178	0257	0997	.786
1.3106	1.2932	4842	7728	0304	1047	.788
1.3046	1.2874	0.95374	8280	0.10351	1097	0.790
1.2987	1.2816	5908	8834	0398	1147	.792
1.2928	1.2759	6443	9390	0445	1197	.794
1.2870	1.2702	6981	9947	0492	1247	.796
1.2813	1.2645	7520	*0507	0539	1297	.798

TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same appears, indicating that next digit

M	M^2	I_A γ 1.37	I_A γ 1.40	I_f γ 1.37	I_f γ 1.40	I_T γ 1.37	I_T γ 1.40
0.800	0.64000	-3.9765	*107	1.7433	968	3.7316	*021
.802	4320	-4.0345	693	776	*322	948	*668
.804	4642	- 936	*291	.8126	684	.8594	*329
.806	4964	- .1540	902	485	*054	.9256	*005
.808	5286	- .2157	526	853	*434	932	*697
0.810	0.65610	-4.2788	*163	1.9230	824	4.0624	*405
.812	5934	- .3432	815	616	*222	.1332	*130
.814	.66260	- .4091	481	2.0012	631	.2057	872
.816	6586	- 764	*162	417	*050	799	*631
.818	6912	- .5452	858	833	*479	.3559	*408
0.820	0.67240	-4.6156	570	2.1259	920	4.4338	*205
.822	7568	- 877	*299	697	*372	.5135	*021
.824	7898	- .7614	*045	.2145	835	952	*857
.826	8228	- .8369	808	606	*310	.6790	*715
.828	8558	- .9141	590	.3078	798	.7648	*593
0.830	0.68890	-4.9932	*390	2.3563	*300	4.8529	*495
.832	9222	-5.0743	*210	.4061	814	.9432	*419
.834	9556	- .1573	*050	572	*343	5.0359	*368
.836	9890	- .2424	911	.5098	886	.1310	*341
.838	.70224	- .3297	794	638	*444	.2286	*341
0.840	0.70560	-5.4192	699	2.6193	*017	5.3289	*367
.842	0896	- .5110	628	764	*607	.4319	*421
.844	1234	- .6052	581	.7351	*214	.5377	*504
.846	1572	- .7019	560	954	*838	.6464	*617
.848	1910	- .8012	564	.8576	*480	.7582	*762
0.850	0.72250	-5.9032	596	2.9216	*141	5.8732	*939
.852	2590	-6.0080	657	874	*822	.9915	*1150
.854	2932	- .1157	747	3.0553	*523	6.1132	*397
.856	3274	- .2265	868	.1252	*246	.2385	*680
.858	3616	- .3405	*021	973	*991	.3676	5002
0.860	0.73960	-6.4577	*207	3.2716	*759	6.5005	*363
.862	4304	- .5784	*429	.3483	*552	.6376	*766
.864	4650	- .7028	687	.4275	*370	.7788	9213
.866	4996	- .8308	983	.5091	*214	.9246	*706
.868	5342	- .9629	*319	935	7086	7.0750	2246'
0.870	0.75690	-7.0990	*697	3.6807	*987	7.2302	*836
.872	6038	- .2395	*119	.7708	*919	.3905	5478
.874	6388	- .3845	*586	.8640	*882	.5562	7175
.876	6738	- .5342	*101	.9604	*879	.7275	*930
.878	7088	- .6889	*667	4.0602	*911	.9046	*744

NACA

SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for γ = 1.37 unless an asterisk to left is increased one unit.]

I_R		I_c		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
1.2756	1.2589	0.98061	*1069	0.10587	1348	0.800
1.2699	1.2534	8604	*1633	0634	1398	.802
1.2642	1.2478	9150	*2198	0681	1448	.804
1.2586	1.2424	9696	*2766	0729	1499	.806
1.2531	1.2369	1.00245	3336	0776	1549	.808
1.2476	1.2315	1.0080	391	0.10824	1600	0.810
1.2421	1.2262	135	448	0872	1650	.812
1.2366	1.2209	190	506	0920	1701	.814
1.2313	1.2156	246	563	0967	1752	.816
1.2259	1.2104	302	621	1015	1803	.818
1.2206	1.2052	1.0358	680	0.11063	1854	0.820
1.2153	1.2000	414	738	1111	1905	.822
1.2101	1.1949	470	796	1159	1956	.824
1.2049	1.1898	527	855	1208	2007	.826
1.1997	1.1847	584	914	1256	2058	.828
1.1946	1.1797	1.0641	973	0.11304	2110	0.830
1.1895	1.1747	698	*033	1352	2161	.832
1.1844	1.1698	755	*092	1401	2212	.834
1.1794	1.1649	813	*152	1449	2264	.836
1.1745	1.1600	871	*212	1498	2315	.838
1.1695	1.1552	1.0929	*272	0.11546	2367	0.840
1.1646	1.1504	987	*333	1595	2418	.842
1.1597	1.1456	.1045	394	1644	2470	.844
1.1549	1.1409	104	454	1693	2522	.846
1.1501	1.1362	162	515	1741	2574	.848
1.1453	1.1315	1.1221	577	0.11790	2626	0.850
1.1406	1.1268	280	638	1839	2678	.852
1.1359	1.1222	340	700	1888	2730	.854
1.1312	1.1177	399	762	1937	2782	.856
1.1266	1.1131	459	824	1987	2834	.858
1.1220	1.1086	1.1519	886	0.12036	2886	0.860
1.1174	1.1042	579	948	2085	2938	.862
1.1128	1.0997	639	*011	2134	2990	.864
1.1083	1.0953	700	*074	2184	3043	.866
1.1038	1.0909	761	*137	2233	3095	.868
1.0994	1.0866	1.1822	*201	0.12283	3148	0.870
1.0950	1.0822	883	*264	2332	3200	.872
1.0906	1.0779	944	*328	2382	3253	.874
1.0862	1.0737	.2006	392	2432	3306	.876
1.0819	1.0694	067	456	2481	3358	.878

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TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	I_A		I_f		I_T	
		γ		γ		γ	
		1.37	1.40	1.37	1.40	1.37	1.40
0.880	0.77440	-7.8488	*285	4.1635	*979	8.0879	2622
.882	7792	-8.0142	959	.2706	4086	.2777	4566
.884	8146	- .1854	*692	.3816	5234	.4743	6580
.886	8500	- .3626	*486	.4967	6424	.6780	8667
.888	8854	- .5462	*345	.6163	*660	.8894	*0833
0.890	0.79210	-8.7366	*272	4.7404	*944	9.1087	3080
.892	9566	- .9341	*271	.8694	*0278	.3364	5413
.894	9924	-9.1392	*346	5.0035	*665	.5731	7838
.896	.80282	- .3522	*502	.1430	3108	.8191	*0359
.898	0640	- .5736	*744	.2883	4610	10.0075	2983
0.900	0.81000	-9.8040	*076	5.4397	6176	10.342	571
.902	1360	-10.0438	*504	.5976	7809	.620	856
.904	1722	- .2938	*034	.7624	9512	.909	*153
.906	2084	- .5544	*672	.9345	*1292	11.212	463
.908	2446	- .8265	*426	6.1144	3153	.528	.787
0.910	0.82810	-11.111	230	6.3025	5099	11.858	*125
.912	3174	- .408	531	.4996	7137	12.204	479
.914	3540	- .719	846	.7062	9274	.566	850
.916	3906	-12.045	176	.9230	*1517	.946	*240
.918	4272	- .387	523	7.1507	3872	13.344	649
0.920	0.84640	-12.746	886	7.3902	6350	13.764	*078
.922	5008	-13.124	269	.6424	8958	14.205	530
.924	5378	- .522	672	.9082	*1708	.669	*007
.926	5748	- .942	*096	8.1888	4611	15.160	509
.928	6118	-14.384	545	.4854	7679	.678	*040
0.930	0.86490	-14.853	*019	8.7995	*0928	16.226	602
.932	6862	-15.348	521	9.1324	4371	.807	*198
.934	7236	- .874	*053	.4860	8030	17.423	830
.936	7610	-16.434	619	.8622	*1921	18.079	502
.938	7984	-17.029	222	10.2632	6069	.778	*218
0.940	0.88360	-17.664	865	10.691	*050	19.523	982
.942	8736	-18.343	553	11.150	524	20.321	800
.944	9114	-19.071	289	.641	*033	21.176	677
.946	9492	- .852	*081	12.170	580	22.096	620
.948	9870	-20.694	933	.740	*169	23.087	636
0.950	0.90250	-21.604	854	13.356	806	24.158	734
.952	0630	-22.589	852	14.024	498	25.318	924
.954	1012	-23.661	937	.751	*250	26.581	*218
.956	1394	-24.829	*121	15.544	*071	27.959	*631
.958	1776	-26.110	417	16.415	971	29.470	*180

SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Continued

as those shown for $\gamma = 1.37$ unless an asterisk to left is increased one unit.]

I_R γ 1.37	I_R γ 1.40	I_c γ 1.37	I_c γ 1.40	$I_{T,e}$ γ 1.37	$I_{T,e}$ γ 1.40	M
1.0776	1.0652	1.2129	521	0.12531	3411	0.880
1.0733	1.0611	191	585	2581	3464	.882
1.0691	1.0569	254	650	2631	3517	.884
1.0649	1.0528	316	715	2681	3570	.886
1.0607	1.0487	379	781	2731	3622	.888
1.0565	1.0446	1.2442	846	0.12781	3676	0.890
1.0524	1.0406	505	912	2831	3729	.892
1.0483	1.0366	568	978	2881	3782	.894
1.0442	1.0326	632	*044	2932	3835	.896
1.0402	1.0286	696	*110	2982	3888	.898
1.0362	1.0247	1.2760	*177	0.13032	3942	0.900
1.0322	1.0208	824	*244	3082	3995	.902
1.0282	1.0169	889	*311	3133	4048	.904
1.0243	1.0130	953	*378	3184	4102	.906
1.0204	1.0092	.3018	446	3234	4155	.908
1.01648	1.00542	1.3083	514	0.13285	4209	0.910
1.01262	1.00164	148	581	3335	4262	.912
1.00879	0.99789	214	650	3386	4316	.914
1.00497	.99415	279	718	3437	4370	.916
1.00119	.99045	345	787	3488	4424	.918
0.99742	0.98677	1.3411	856	0.13538	4477	0.920
.99368	.98311	478	925	3589	4531	.922
.98998	.97947	544	994	3640	4585	.924
.98629	.97586	611	*063	3691	4639	.926
.98262	.97228	678	*133	3742	4693	.928
0.97898	0.96872	1.3745	*203	0.13794	4747	0.930
.97537	.96518	812	*273	3845	4801	.932
.97176	.96166	880	*344	3896	4855	.934
.96820	.95817	948	*414	3947	4910	.936
.96464	.95469	.4016	485	3999	4964	.938
0.96112	0.95124	1.4084	556	0.14050	5018	0.940
.95761	.94781	153	628	4101	5072	.942
.95413	.94440	221	699	4153	5127	.944
.95068	.94102	290	771	4204	5181	.946
.94724	.93765	359	843	4256	5236	.948
0.94382	0.93431	1.4429	916	0.14307	5290	0.950
.94042	.93098	498	988	4359	5345	.952
.93705	.92769	568	*061	4411	5399	.954
.93370	.92441	638	*134	4462	5454	.956
.93037	.92115	708	*207	4514	5509	.958

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TABLE I - FUNCTIONS OF MACH NUMBER REQUIRED IN NUMERICAL
IN ROTOR COOLANT

[Digits to left of those shown for γ = 1.40 are same
appears, indicating that next digit

M	M^2	I_A		I_f		I_T	
		γ		γ		γ	
		1.37	1.40	1.37	1.40	1.37	1.40
0.960	0.92160	- 27.519	844	17.372	962	31.132	884
.962	2544	- 29.076	420	18.432	*059	32.970	*769
.964	2930	- 30.806	*173	19.610	*278	35.013	864
.966	3316	- 32.740	*131	20.928	*642	37.298	*207
.968	3702	- 34.917	*335	22.412	*177	39.870	*844
0.970	0.94090	- 37.383	833	24.094	918	42.786	*834
.972	4478	- 40.203	688	26.018	909	46.120	*253
.974	4868	- 43.456	982	28.240	*208	49.968	51.199
.976	5258	- 47.252	826	30.833	*891	54.459	*804
.978	5648	- 51.739	*370	33.899	35.064	59.768	61.248
0.980	0.96040	- 57.123	822	37.580	*873	66.142	*784
.982	6432	- 63.704	*486	42.081	*530	73.933	75.773
.984	6826	- 71.932	*818	47.709	49.354	83.675	85.763
.986	7220	- 82.510	*530	54.948	56.844	96.203	98.610
.988	7614	- 96.615	*813	64.603	66.836	112.910	115.742
0.990	0.98010	-116.36	117.81	78.122	80.827	136.30	139.73
.992	8406	-145.99	147.81	98.407	101.817	171.40	175.72
.994	8804	-195.36	197.81	132.220	136.808	229.90	235.71
.996	9202	-294.11	297.80	199.854	206.798	346.91	355.70
.998	9600	-590.35	597.80	402.778	416.791	697.96	715.69
1.000	1.0000	∞	∞	∞	∞	∞	∞

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SOLUTIONS OF EQUATIONS FOR ONE-DIMENSIONAL GAS FLOW PASSAGES - Concluded

as those shown for $\gamma = 1.37$ unless an asterisk to left is increased one unit.]

I_R		I_c		$I_{T,e}$		M
γ		γ		γ		
1.37	1.40	1.37	1.40	1.37	1.40	
0.92706	0.91791	1.4779	*281	0.14566	5563	0.960
.92377	.91469	849	*354	4618	5618	.962
.92050	.91149	920	*428	4670	5673	.964
.91725	.90831	991	*502	4722	5728	.966
.91402	.90515	.5062	577	4774	5783	.968
0.91081	0.90201	1.5134	651	0.14826	5838	0.970
.90762	.89889	206	726	4878	5893	.972
.90445	.89578	278	802	4930	5948	.974
.90130	.89270	350	877	4982	6003	.976
.89818	.88964	422	952	5035	6058	.978
0.89506	0.88660	1.5495	*028	0.15087	6113	0.980
.89197	.88357	568	*104	5139	6168	.982
.88889	.88056	641	*181	5192	6223	.984
.88584	.87757	715	*257	5244	6279	.986
.88280	.87460	788	*334	5296	6334	.988
0.87978	0.87165	1.5862	*411	0.15349	6389	0.990
.87678	.86871	936	*488	5401	6445	.992
.87380	.86579	.6010	566	5454	6500	.994
.87084	.86289	085	644	5506	6556	.996
.86789	.86001	160	722	5559	6611	.998
0.86496	0.85714	1.6234	800	0.15612	6667	1.000

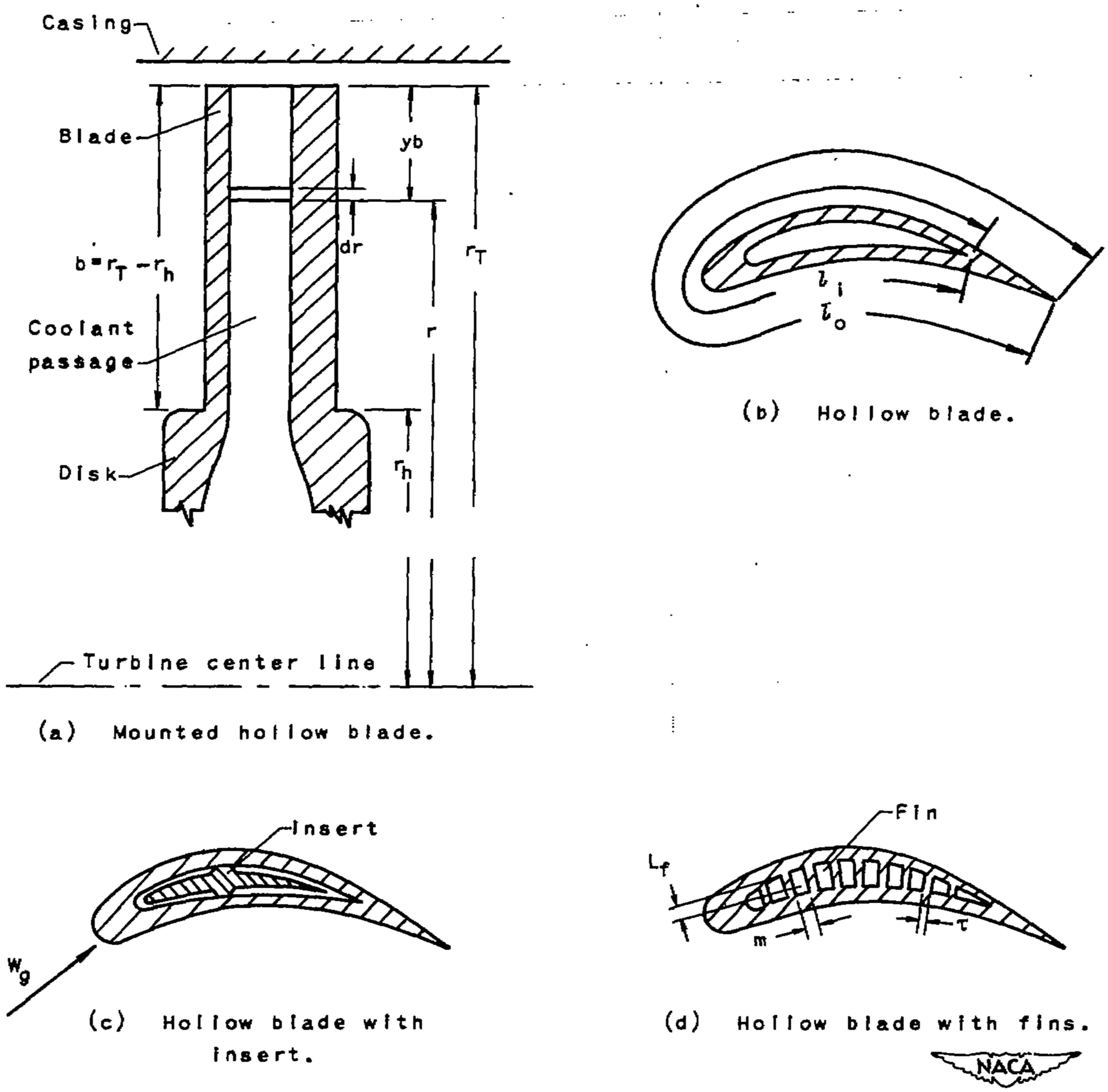
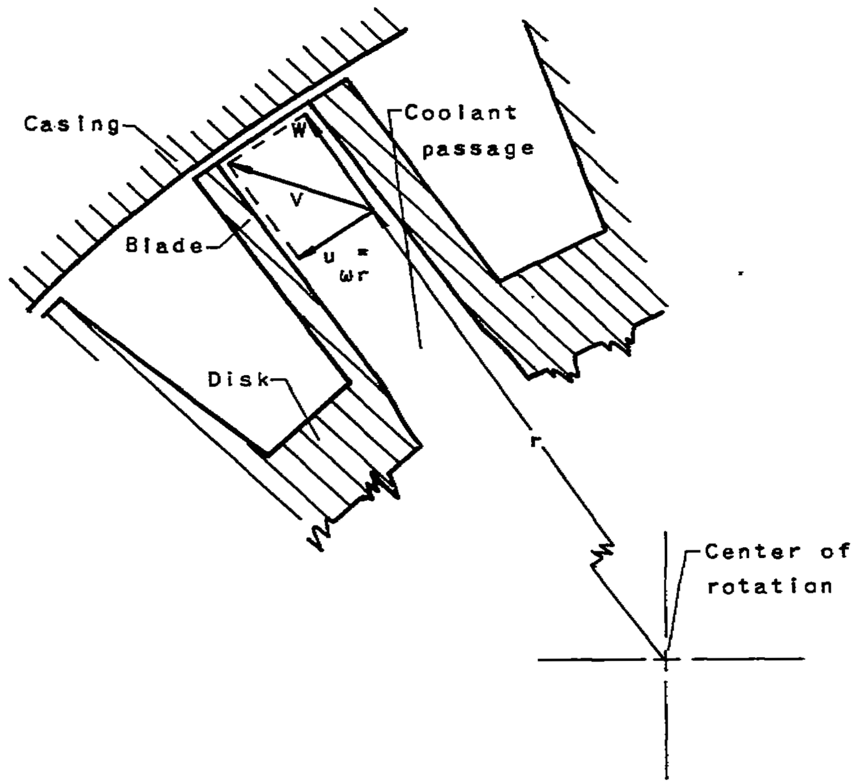
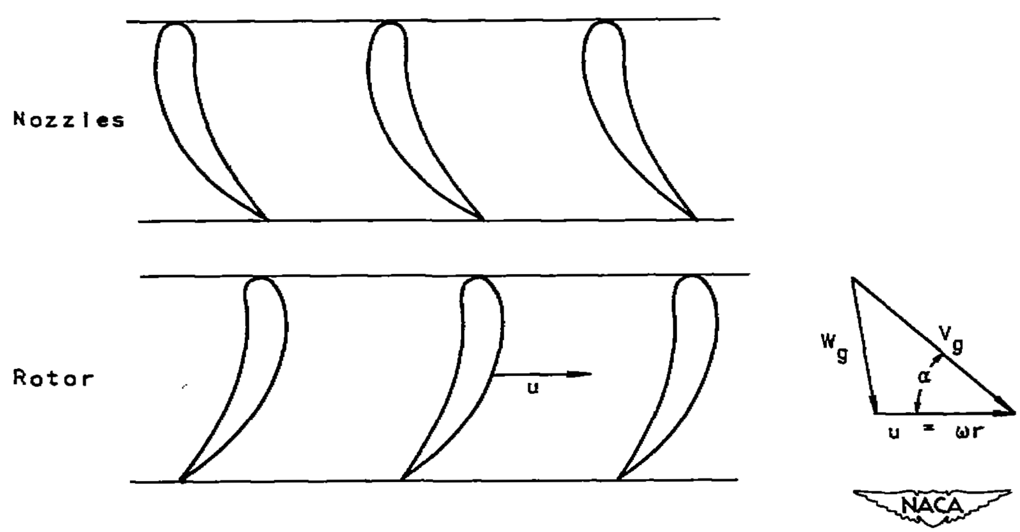


Figure 1. - Pertinent blade dimensions and several blade configurations.



(a) Coolant velocities.



(b) Combustion-gas velocities at entrance to rotor stage.

Figure 2. - Fluid-velocity vector diagrams.

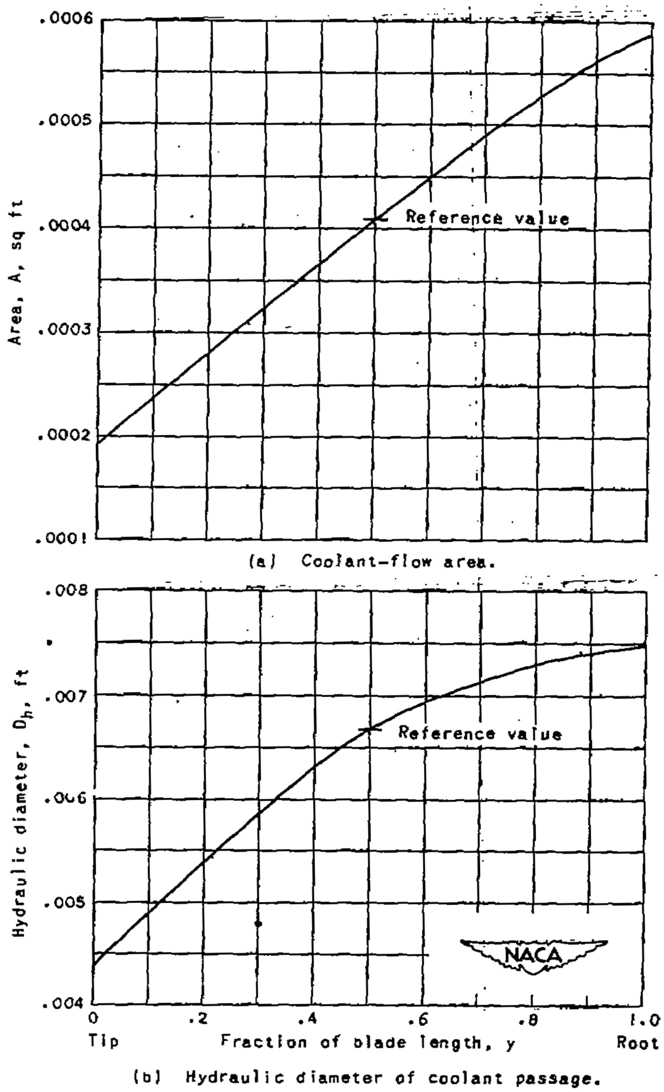
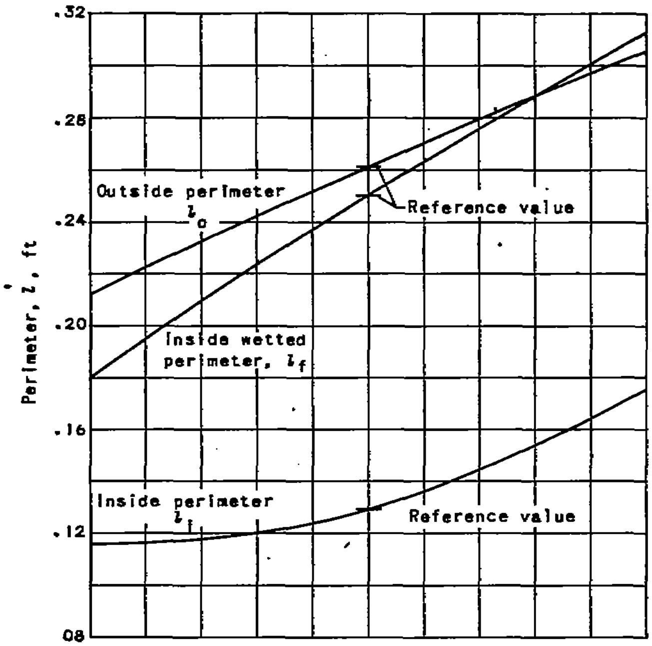
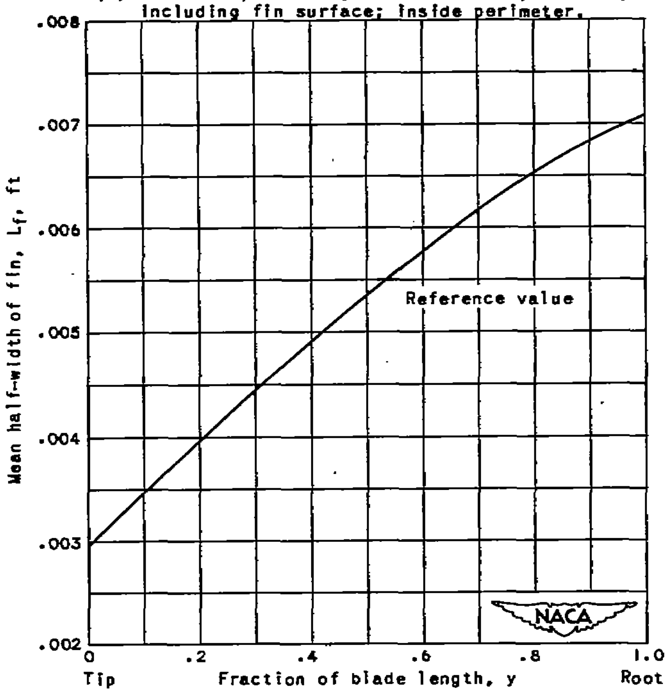


Figure 3. - Variation from tip to root of finned-hollow-blade data used in numerical example.

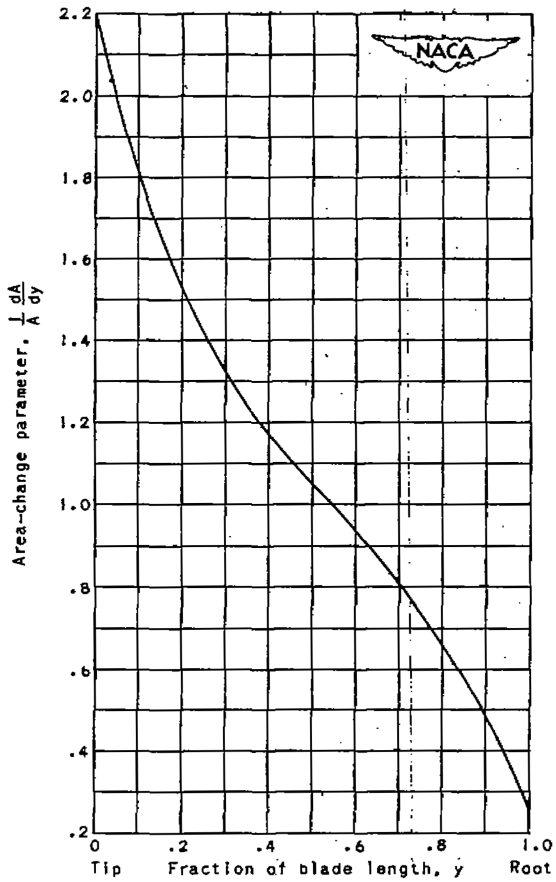


(c) Outside perimeter; inside wetted perimeter, including fin surface; inside perimeter.



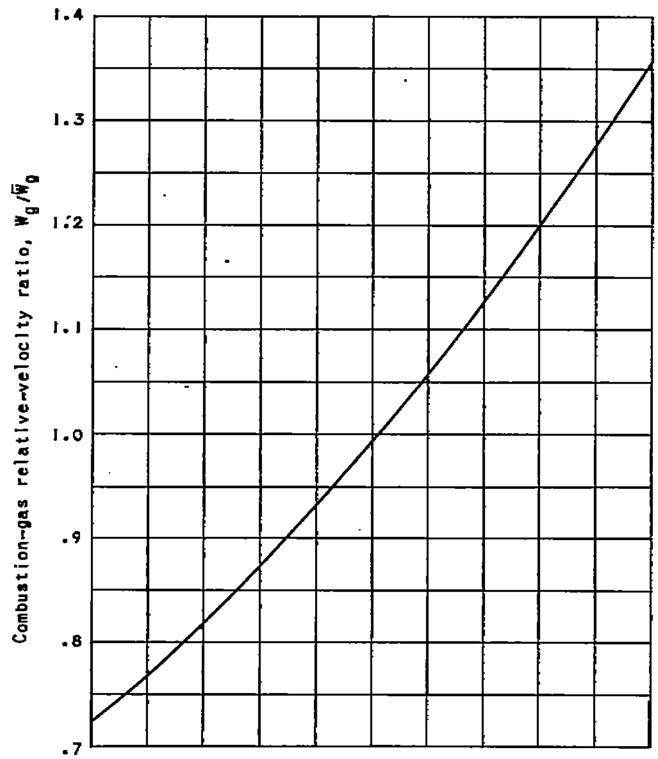
(d) Mean half-width of fins.

Figure 3. - Continued. Variation from tip to root of finned-hollow-blade data used in numerical example.

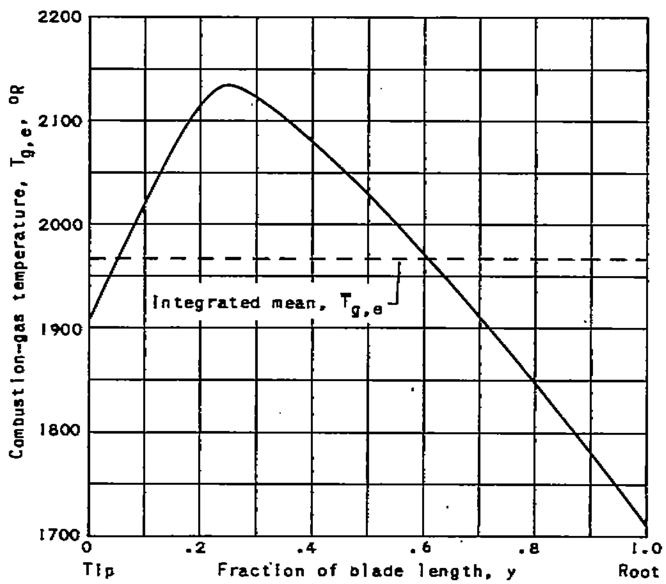


(e) Area-change parameter, $\frac{1}{A}\frac{dA}{dy}$.

Figure 3. - Continued. Variation from tip to root of finned-hollow-blade data used in numerical example.



(f) Combustion-gas relative-velocity ratio.



(g) Combustion-gas temperature.

$$\overline{T}_{g,e} = \int_0^1 T_{g,e}\,dy$$

Figure 3. – Concluded. Variation from tip to root of finned-hollow-blade data used in numerical example.

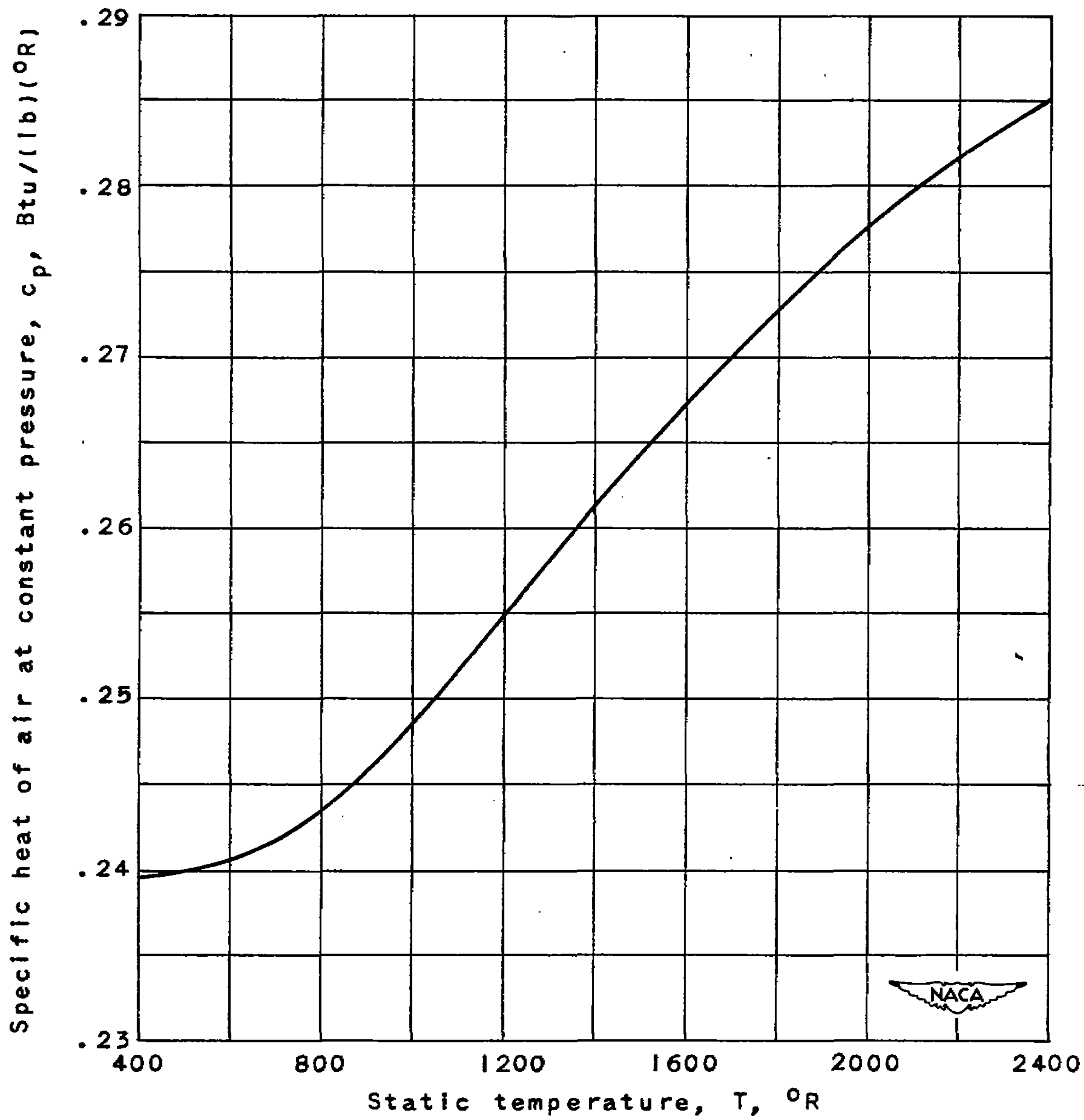


Figure 4. - Variation of specific heat of air at constant pressure with absolute temperature (reference 15).

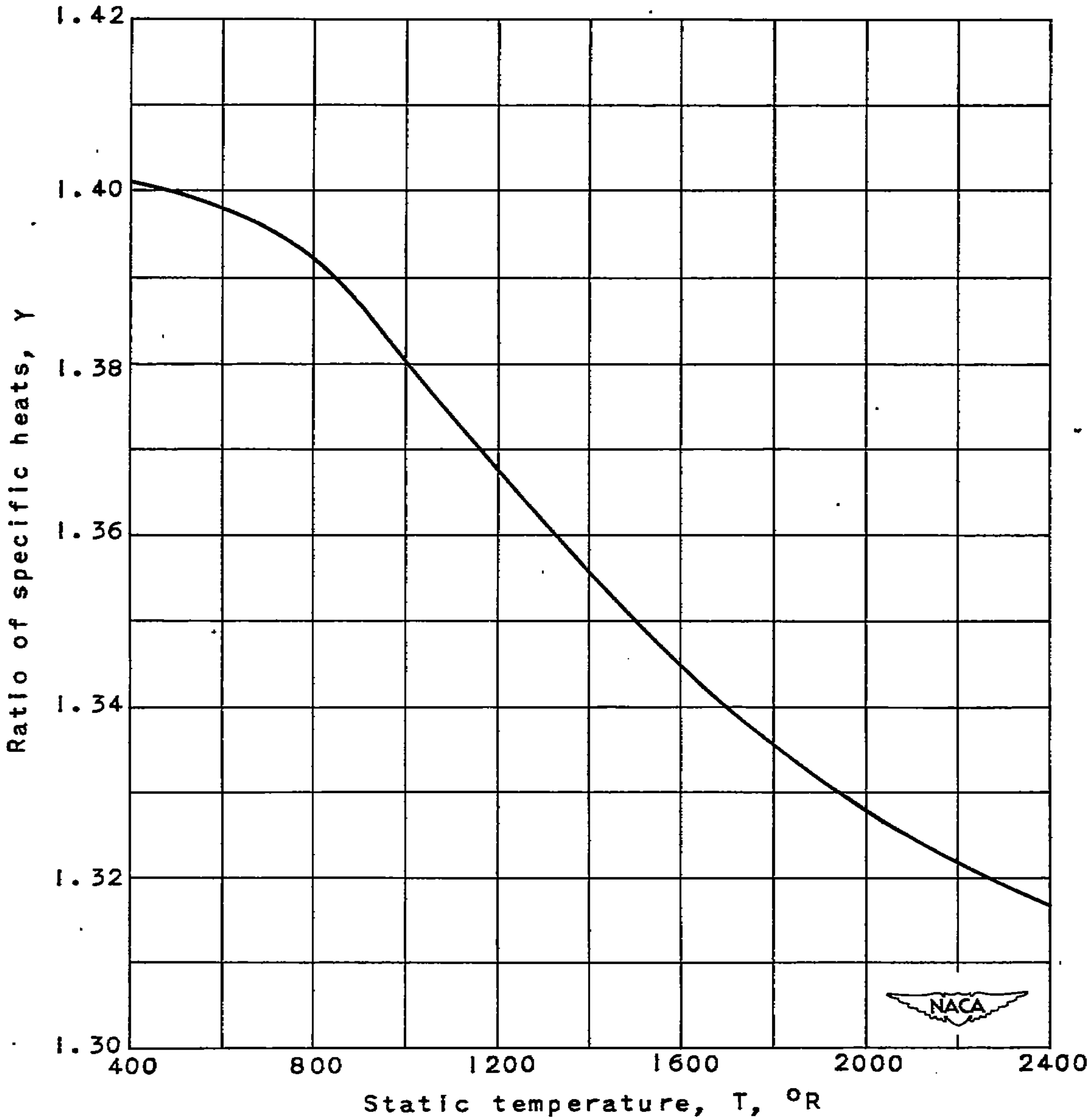


Figure 5. - Variation of ratio of specific heats of air with absolute temperature (reference 15).

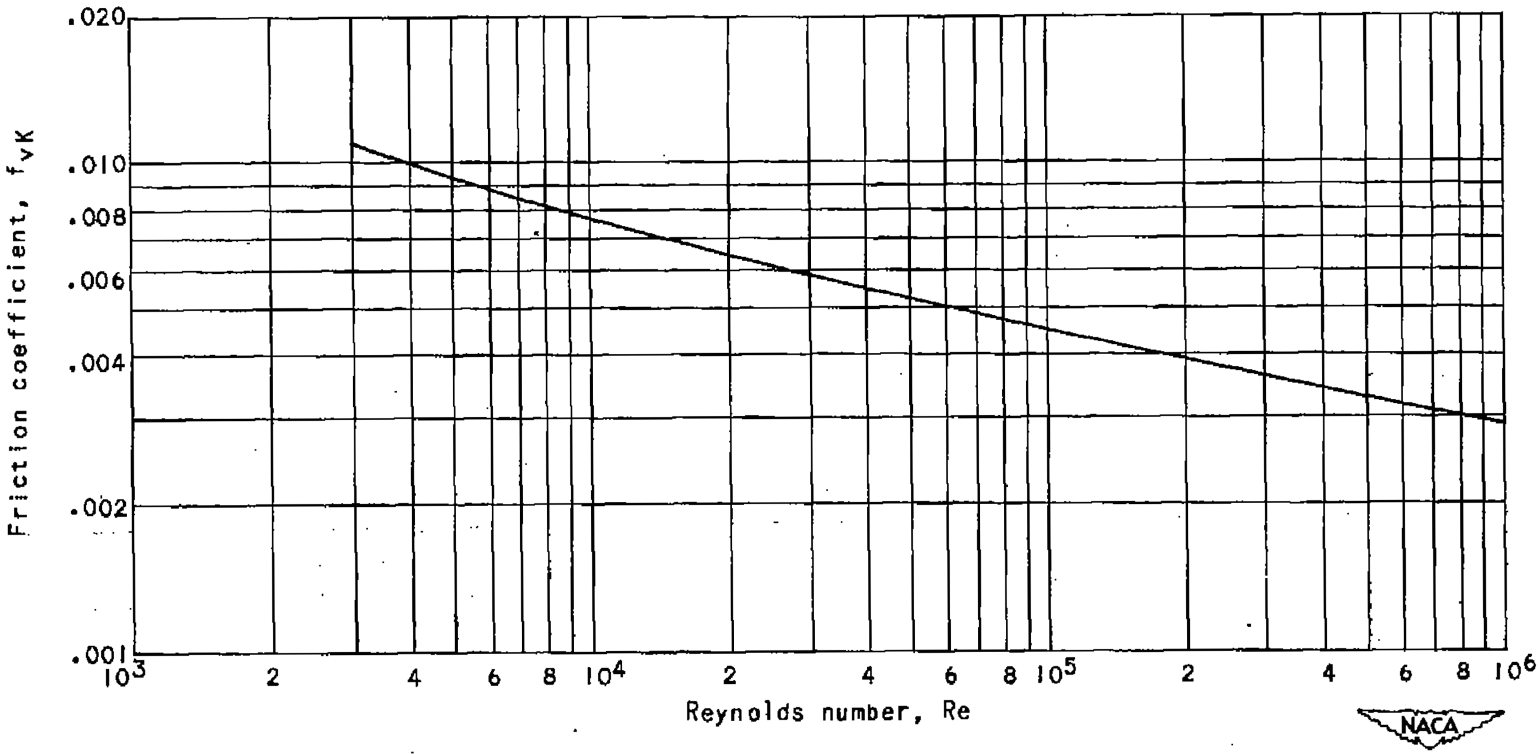


Figure 6. - Variation of friction coefficient with Reynolds number for flow in smooth pipes (equation (101)).

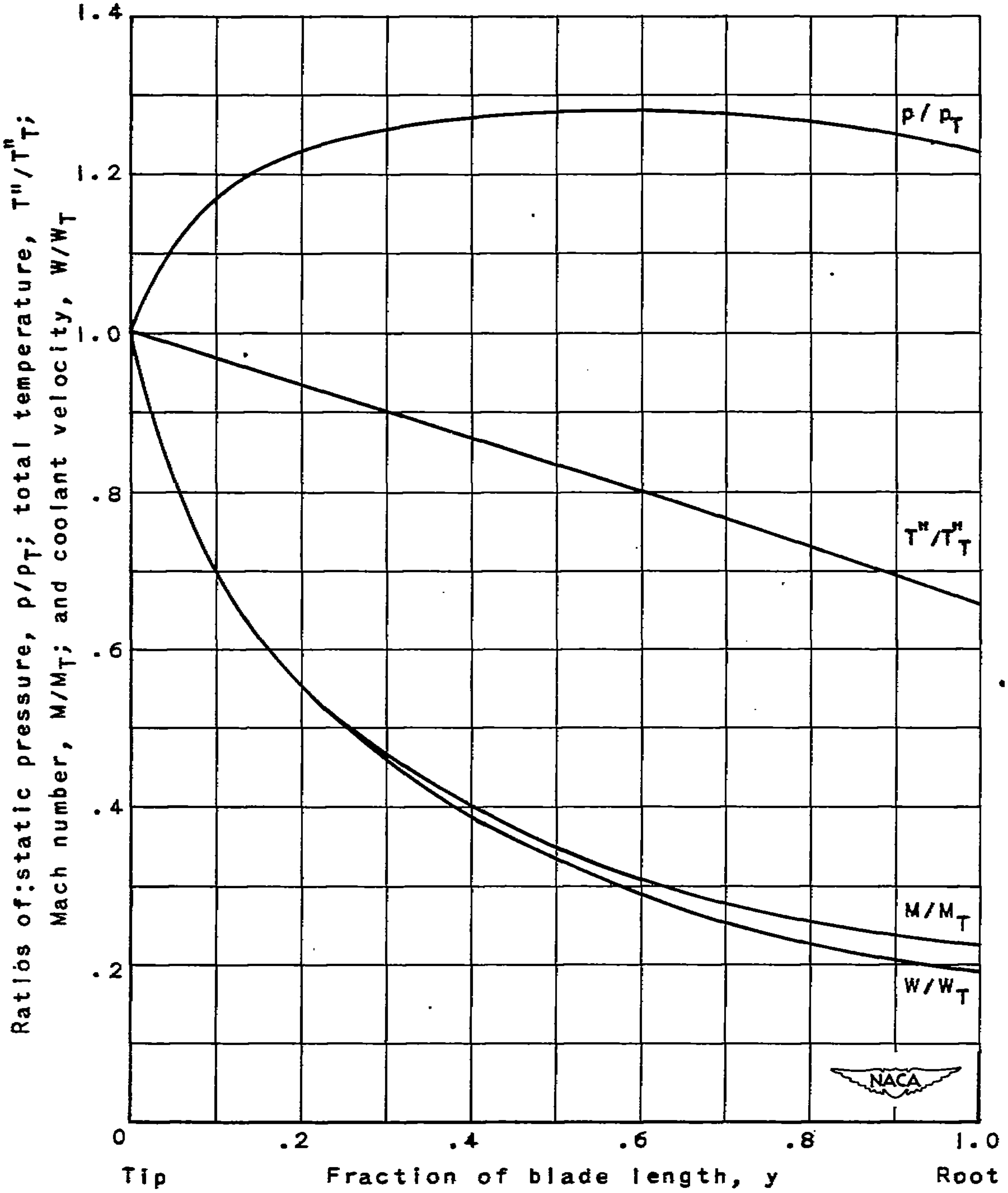


Figure 7. - Variation of static pressure, total temperature, Mach number, and velocity of coolant from tip to root as determined from open-form solution of momentum and energy equations.

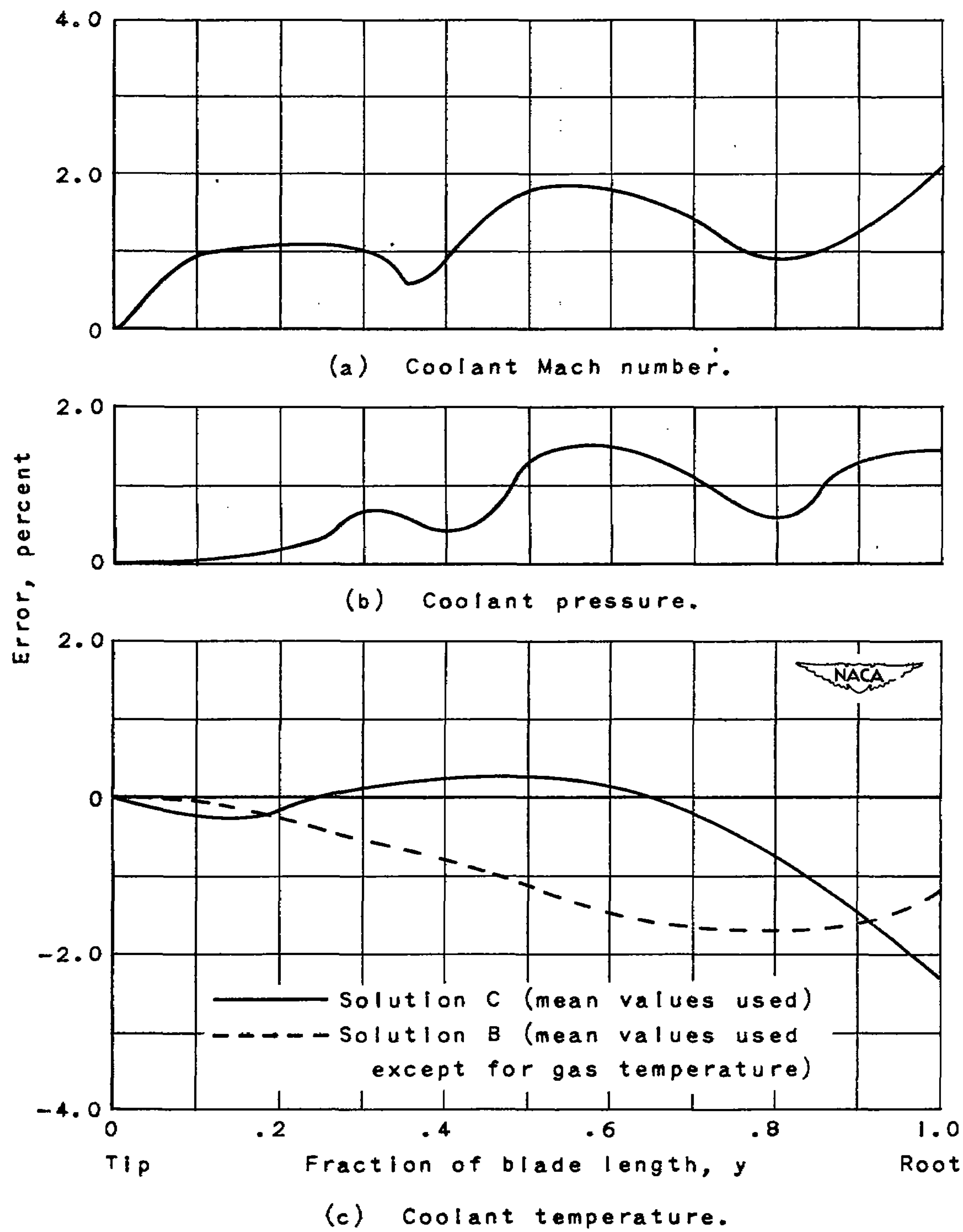


Figure 8. - Error in simplified solutions for Mach number, pressure, and temperature.

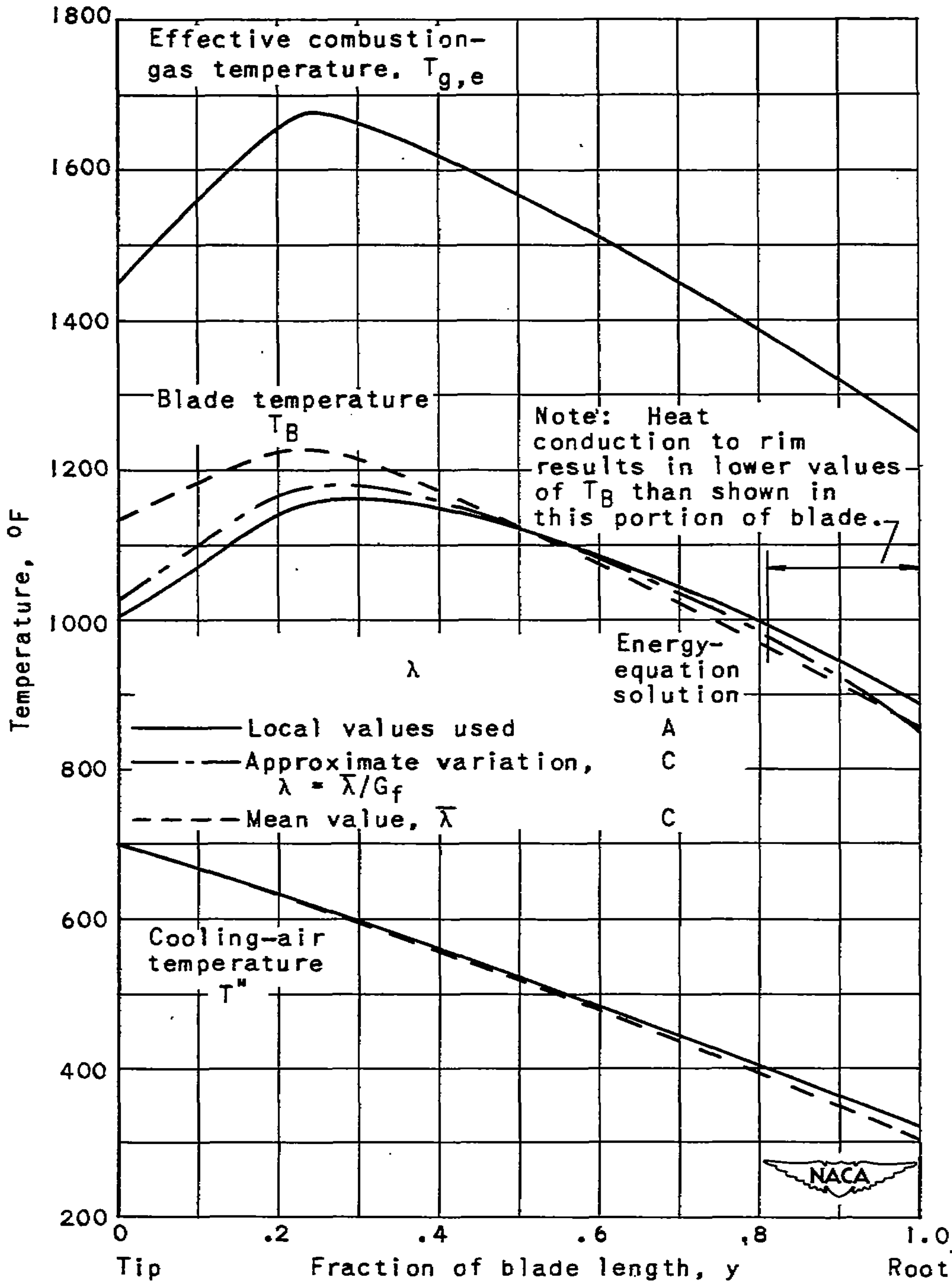


Figure 9. - Computed cooling-air total and blade-metal temperature distribution computed from relation $T_B = (\lambda T_{g,e} + T'')/(1 + \lambda)$.